Current problems in few-nucleon physics: selected examples

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2026 Prof. A.N.Mitra Memorial Lecture Series

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Outline

Introduction

Nuclear interaction – 3NF and new ideas

Uncertainty quantification

Emulators

Electroweak processes

Relativistic approach

Search for quantum entanglement in nuclear systems

Summary

Introduction

We (group at the Jagiellonian University and collaborators) are interested in:

- Nucleon-deuteron scattering at beam E_N in [0-250] MeV

  $N+d \rightarrow N+d, \quad N+d \rightarrow N+N+N$

- Electroweak processes with ^2H,^3H or ^3He: e.g.

  $e+{}^2H \rightarrow e'+n+p, \quad \gamma+{}^3H \rightarrow n+n+p, \quad \nu+d \rightarrow \nu+n+p, \quad \ldots$

and aim to describe these processes starting from model 2N and 3N forces and model of currents

- That aim is in line of broader efforts to answer more general question: How accurately can we describe nuclear phenomena without referring to quark-gluon degrees of freedom?

- In some sense the above program is a continuation of works by prof. Mitra

The legacy of Professor Asoke Nath Mitra

- In our context, it is worth recalling that Prof. Mitra developed the research on three-nucleon systems with separable potentials:

1.B *Nuclear Physics* **32** (1962) 529—542; © *North-Holland Publishing Co., Amsterdam*

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THREE-BODY PROBLEM WITH SEPARABLE POTENTIALS
(I) Bound States

A. N. MITRA
Dept. of Physics, University of Delhi, Delhi-6, India

Received 2 January 1962

PHYSICAL REVIEW VOLUME 131, NUMBER 3 1 AUGUST 1963

Three-Body Problem with Separable Potentials. II. *n-d* Scattering*

A. N. MITRA,†
Department of Physics, Indiana University, Bloomington, Indiana
AND
V. S. BHASIN
Department of Physics, Delhi University, Delhi, India
(Received 29 March 1963)

PHYSICAL REVIEW VOLUME 137, NUMBER 2B 25 JANUARY 1965

Neutron-Deuteron Scattering at Low Energies*

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Department of Physics and Astrophysics, University of Delhi, Delhi, India
AND
G. L. SCHRENK† AND A. N. MITRA‡
Department of Physics, Indiana University, Bloomington, Indiana
(Received 1 September 1964)

ACKNOWLEDGMENTS

Most of this work was done when two of us (ANM and GLS) were at Indiana University during the session 1963. We are grateful to Professor E. J. Konopinski for the excellent facilities of the physics department and in particular a good deal of running time on the IBM 709 computer at Indiana University, without which this work would not have been possible. We also acknowledge helpful discussion with B. S. Bhakar.

- Dependence of binding energy on interaction range and other parameters
- Restriction to s and p waves
- Role of the tensor force

The legacy of Professor Asoke Nath Mitra

- Relativistic description

VOLUME 15, NUMBER 25 PHYSICAL REVIEW LETTERS 20 DECEMBER 1965

RELATIVISTIC CORRECTIONS TO THE TRITON BINDING ENERGY

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and

B. S. Bhakar and A. N. Mitra
Department of Physics and Astrophysics, University of Delhi, Delhi, India
(Received 5 November 1965)

PHYSICAL REVIEW D VOLUME 1, NUMBER 12 15 JUNE 1970

Relativistic Corrections to the Three-Body Problem

H. JACOB, V. S. BHASIN, AND A. N. MITRA
Department of Physics and Astrophysics, University of Delhi, Delhi-7, India
(Received 15 January 1970)

- Trineutron

VOLUME 16, NUMBER 12 PHYSICAL REVIEW LETTERS 21 MARCH 1966

EXISTENCE OF THE TRINEUTRON

A. N. Mitra
Department of Physics and Astrophysics, University of Delhi, Delhi, India
and
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Centre for Advanced Study in Theoretical Physics and Astrophysics, University of Delhi, Delhi, India
(Received 15 February 1966)

→

^{3}n more probable than ^{2}n

Prof. Mitra emphasized the role of 3n to study the n-n interaction

… but again simple potentials used and too low experimental accuracy.

- Beyond 3N system

PHYSICAL REVIEW VOLUME 140, NUMBER 5B 6 DECEMBER 1965

Faddeev Formalism for Four-Particle Systems*

A. N. MITRA,† J. GILLESPIE,‡ AND R. SUGAR‡
National Science Foundation 1965 Summer Institute for Theoretical Physics, University of Wisconsin, Madison, Wisconsin

AND

NARGIS PANCHAPAKESAN
Department of Physics, University of Delhi, Delhi, India
(Received 28 July 1965)

PHYSICAL REVIEW VOLUME 153, NUMBER 4 20 JANUARY 1967

Electromagnetic Form Factors of H^3 and He^3 with Realistic Potentials

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AND

B. S. BHAKAR AND A. N. MITRA
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(Received 22 August 1966)

Nd scattering

Our standard method to calculate transition amplitude for 3N scattering is to solve:

- the Schrodinger equation -> deuteron

$$\left(H_0 + V\right)\Psi_d = E_d \Psi_d$$

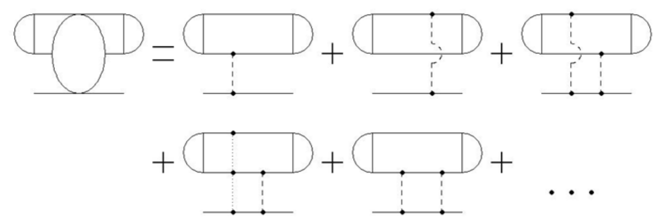
- the Lippmann-Schwinger equation -> t-matrix

$$t(E) = V + VG_0(E)V + VG_0VG_0(E)V + \ldots = V + VG_0 t(E)$$

- the Faddeev equation -> auxiliary operator T

$$T\varphi = tP\varphi + (1 + tG_0)V_{123}^{(1)}(1+P)\varphi + tPG_0T\varphi + (1+tG_0)V_{123}^{(1)}(1+P)G_0T\varphi$$

where ϕ is the deuteron wave function times free nucleon state



- Compute amplitudes for :

elastic scattering: $U = PG_0^{-1} + V_{123}^{(1)}(1+P)\varphi + PT + V_{123}^{(1)}(1+P)G_0T$

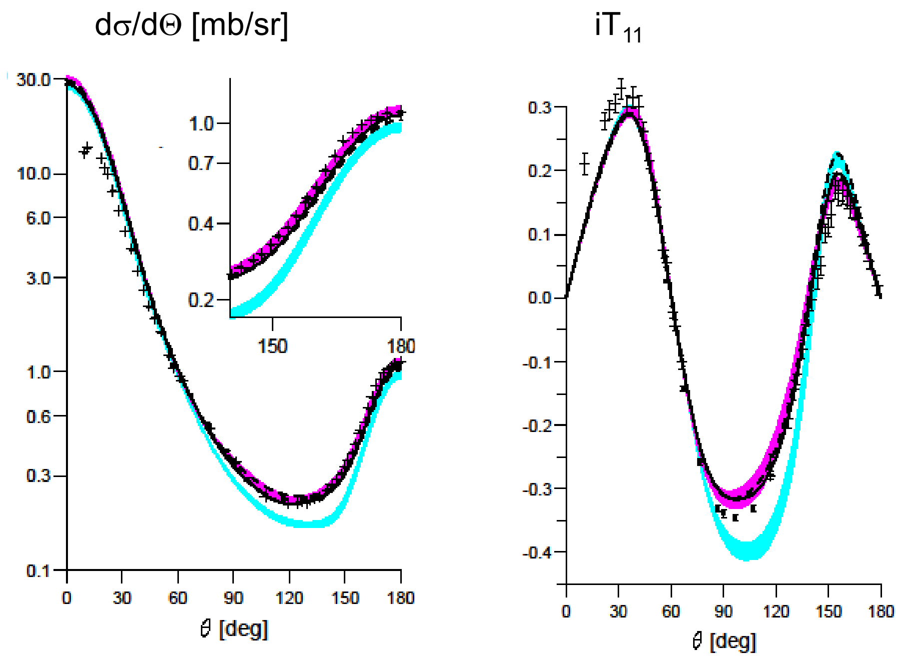
deuteron breakup: $U_0 = (1+P)T$

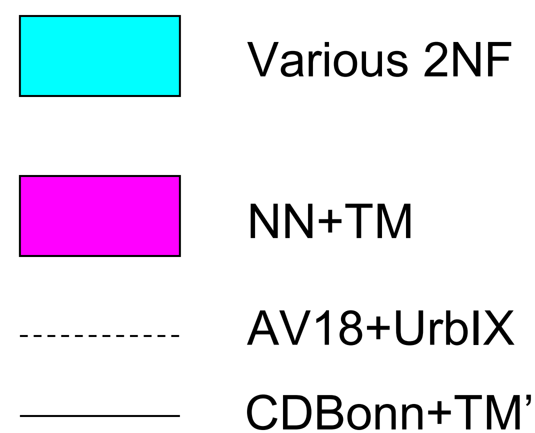
Nuclear interaction

- 1935 meson theory begins with Yukawa potential
- One-boson exchange
- Many-bosons exchange BonnB (CDBonn)
- Important role of many-body interactions

Elastic Nd scattering at E=135 MeV

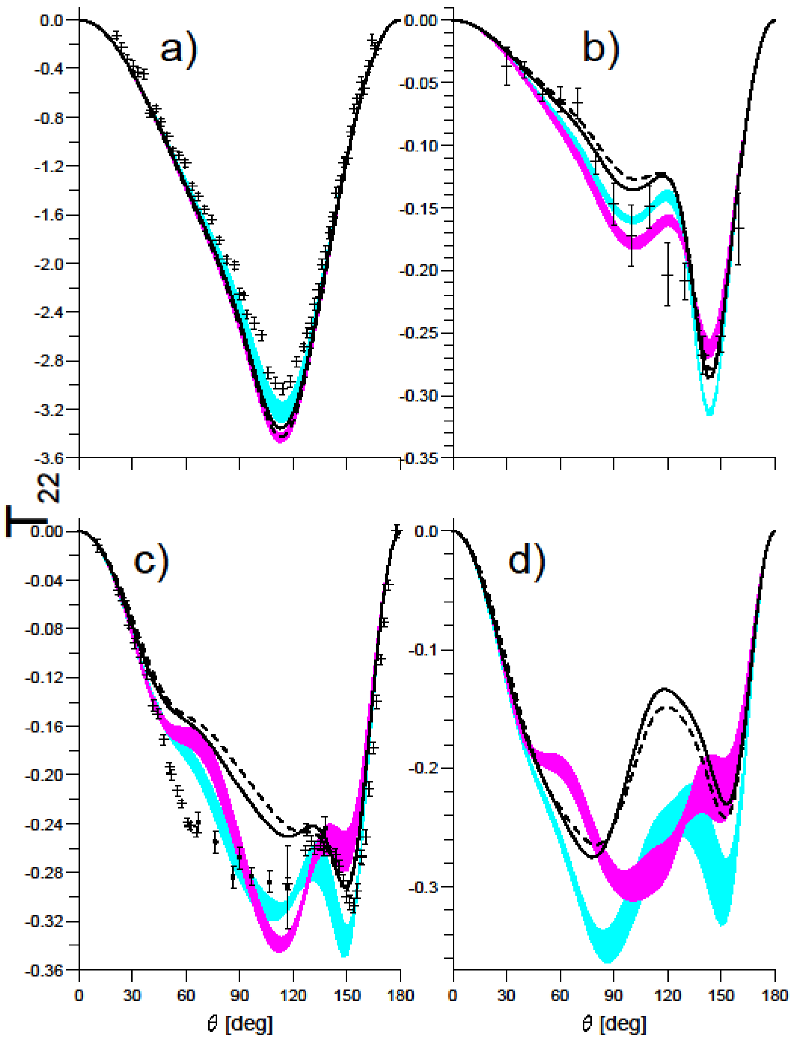
Exp.
H.Sakai, et al., Phys. Rev. Lett. 84 (2000) 5288
N.Sakamoto et al., Phys. Lett. B 367 (1996) 60





H.Witała et al., Phys. Rev. C63 (2001) 024007

Nd elastic scattering



a) E=3 MeV
b) E=65 MeV
c) E=135 MeV
d) E=190 MeV

Various 2NF

NN+TM

AV18+UrbIX

CDBonn+TM’

H.Witała et al., Phys. Rev. C63 (2001) 024007

Chiral interactions

- Started with S.Weinberg's seminal papers in 1990, 1991
- E. Epelbaum, W. Glöckle, Ulf-G. Meißner, Nucl.Phys. A637 (1998) 107-134 – chiral expansion leads to energy independent potential

- Currently we use the chiral SMS potential:

2NF up to N4LO+ (P. Reinert, H. Krebs, E. Epelbaum, Eur. Phys. A54 (2018) 86)
3NF up to N2LO (P.Maris, E.Epelbaum et al., Phys. Rev. C 103 (2021) 054001)

- Intensive ongoing work to complete N3LO 3NF within the LENPIC Collaboration (E.Epelbaum, H.Krebs, K.Hebeler, A.Nogga, K.Topolnicki, and others)

- Examples of other developments:
  1. L. Girlanda, A. Kievsky, and M. Viviani, Phys. Rev. C 84, 014001 (2011).,
  2. L. Girlanda, A. Kievsky, and M. Viviani, Phys. Rev. C 102, 019903(E) (2020).
  3. M. Piarulli, L. Girlanda, R. Schiavilla, A. Kievsky, A. Lovato, L. E. Marcucci, S. C. Pieper, M. Viviani, and R. B. Wiringa, Phys. Rev. C 94, 054007 (2016)
  4. O.Thim, A.Ekström, Ch.Forseen, Phys. Rev. C 112 (2025) 064008

- Review paper: I.Tews et al. Nuclear Forces for Precision Nuclear Physics: A Collection of Perspectives, Few-Body Syst 63 (2022) 67

Chiral interactions

- Dependence on regularization scheme and parameters
- 2N sector

TABLE 1 | χ^2/datum for the description of the neutron-proton and proton-proton scattering data at various orders in the chiral expansion for $\Lambda = 450$ MeV.

E_{lab} bin	$LO_{(3)}$	$NLO_{(10)}$	$N^2LO_{(10)}$	$N^3LO_{(22)}$	$N^4LO_{(23)}$	$N^4LO^+{}_{(27)}$
Neutron–proton scattering data						
0–100	73	2.2	1.2	1.07	1.07	1.07
0–200	62	5.4	1.7	1.09	1.08	1.06
0–300	75	14	4.2	2.01	1.16	1.06
Proton-proton scattering data						
0–100	2290	10	2.2	0.90	0.88	0.86
0–200	1770	90	37	1.99	1.42	0.95
0–300	1380	90	41	3.43	1.67	1.00

The numbers in brackets after the order indicate the number of parameters entering the neutron-proton and proton-proton potentials.

Table from: Epelbaum E, Krebs H and Reinert P (2020) High-Precision Nuclear Forces From Chiral EFT: State-of-the-Art, Challenges, and Outlook. Front. Phys. 8:98. doi: 10.3389/fphy.2020.00098

Chiral interactions

- Dependence on regularization scheme and parameters
- Light nuclei (SRG evolved potential)

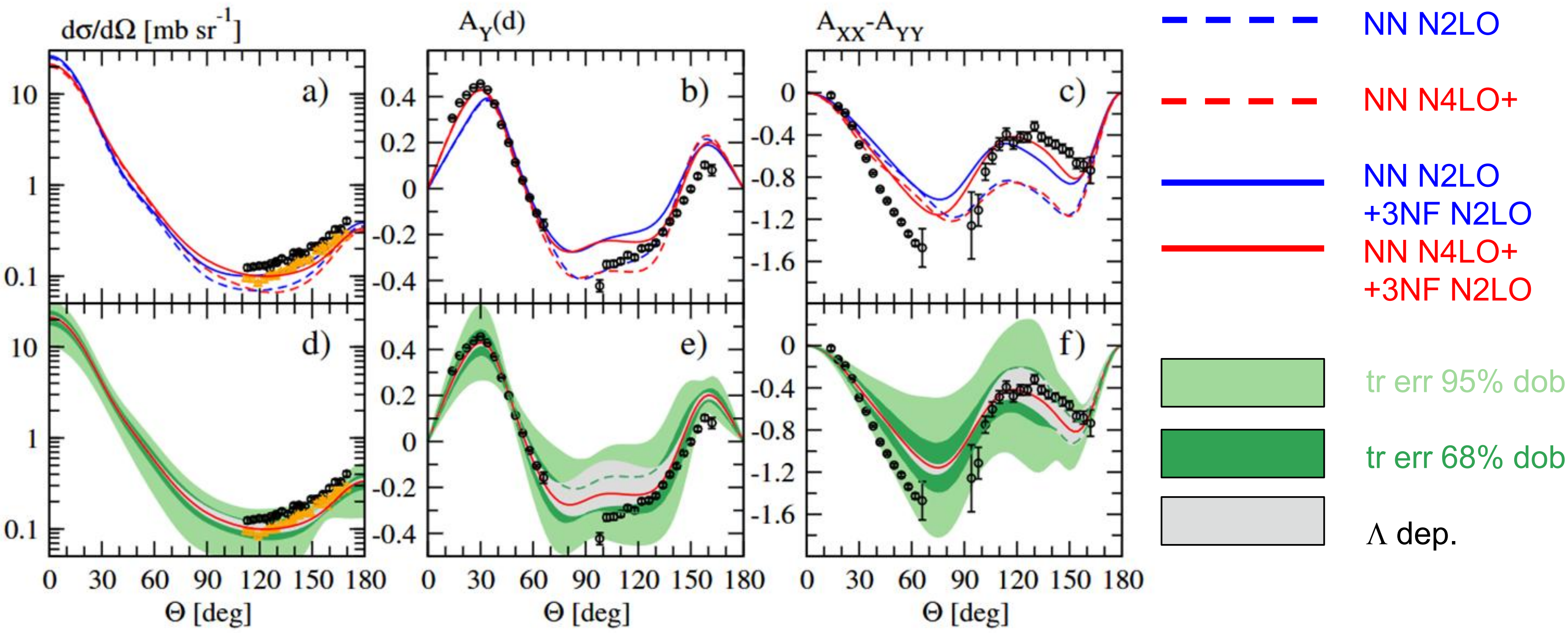
	^{4}He (0^+)	^{14}O (0^+)	
LO	−49.73(0.01)(*)	−152.2(0.7)(*)	Λ=450 MeV
NLO	−29.37(0.01)(3.6)	−98.1(0.7)(32.)	
N^2LO	−28.53(0.01)(1.0)	−113.1(0.4)(11.)	
N^3LO	−28.38(0.01)(1.0)	−99.8(0.8)(11.)	
N^4LO	−28.29(0.01)(1.0)	−99.0(0.8)(11.)	
N^4LO^+	−28.29(0.01)(1.0)	−99.2(0.8)(11.)	
LO	−51.17(0.01)(*)	−148.2(0.9)(*)	Λ=500 MeV
NLO	−28.12(0.01)(3.6)	−89.6(0.6)(39.)	
N^2LO	−28.63(0.01)(1.2)	−116.9(0.4)(13.)	
N^3LO	−28.45(0.01)(1.2)	−103.3(0.8)(13.)	
N^4LO	−28.31(0.01)(1.2)	−102.0(0.9)(13.)	
N^4LO^+	−28.30(0.01)(1.2)	−101.9(0.9)(13.)	
Expt.	−28.296	−98.7	

value (extrapolation)(truncation error)

P.Maris et al., Phys. Rev. C106 (2022) 064002

Chiral interactions

- Dependence on regularization scheme and parameters
- Nucleon-deuteron scattering



E_{lab} = 200 MeV

P.Maris et al., Phys. Rev. C106 (2022) 064002

Other ideas

- Explicit inclusion of Δ d.o.f. (A.Deltuva, H.Krebs, M.Piarulli et al..,others).

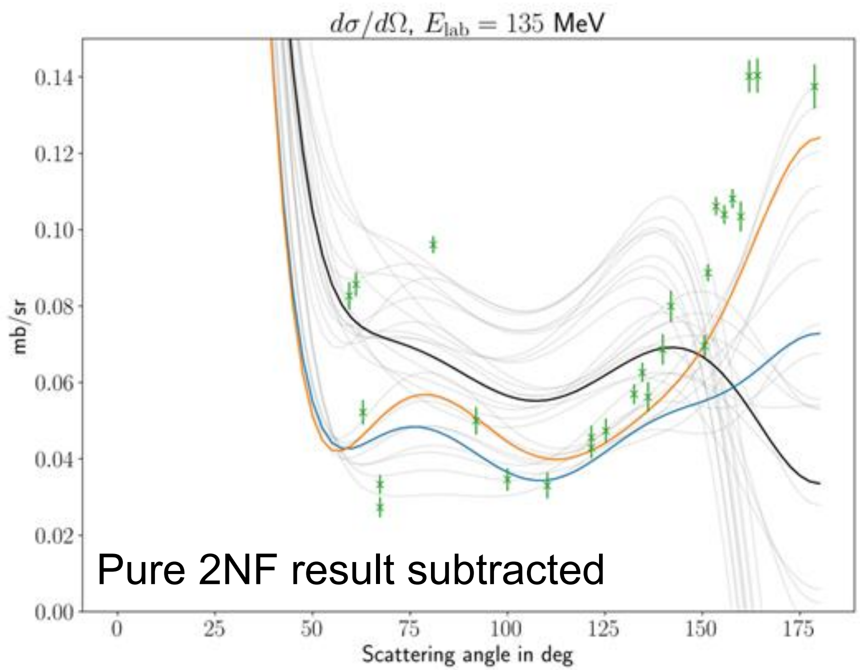
- Lattice QCD – nuclear, i.e. nucleon-nucleon interaction is still under development. First attempts at unphysical pion mass failed even in predicting deuteron properties. Not easy to go to higher partial waves, but still very promising approach.

- Optimized forces: there are ongoing attempts to develop optimized nuclear forces in order to describe specific part of nuclear phenomena, like nuclear matter (e.g. Daejeon potential).
    - One must be careful when using that forces beyond their field of application.
    - AI used during optimalization – often used as one more interpolation or fitting method. But what can we learn from that?
    - Message for a young generation: try to use AI to obtain real qualitative progress.

- Phenomenological potentials: there are propositions on the market to build purely phenomenological interactions e.g. three-nucleon interaction in form of polynomials. Let's not go that way!

Off-shell Low-Energy Constants

- Chiral EFT 2N potential at N3LO comes with 3 off-shell LECs, which can't be determined from 2N system.
- But using unitary transformation of Hamiltonian they can be replaced by equivalent 3NF at N4LO



off-shell LEC = 0,

off-shell LECs fitted with NN N4LO+

off-shell LECs fitted with NN N4LO+ + 3NF N2LO

off-shell LECs in acceptable range

courtesy of Sven Heihoff (Ruhr Universität, Bochum, prof. E.Epelbaum group)

Nd breakup

- We studied, using various combinations of NN and 3N forces, the deuteron breakup reaction n+d→n+n+p, performing global search over the available phase space
- Five independent variables: θ_1, ϕ_1=0, θ_2 , ϕ_2 ,and S
- We use for θ_1, θ_2 , ϕ_2 grids in range 2.5°-177.5° step 5° and for S step 0.5 MeV, which results in approx. 5*10^6 kinematical configurations
- For each (θ_1, θ_2) we look for **maximum** of given effect over $\phi_{12}=\phi_2-\phi_1$ and S(=E_1,E_2)

- We are interested in:

1. Dependence of predictions on the regulator parameter Λ
2. Dependence of predictions on the order of chiral force – here on 2NF
3. Role of the 3N interaction

Nd breakup

The SMS potential is given for a few values of regulator Λ=400, 450, 500, and 550 MeV.

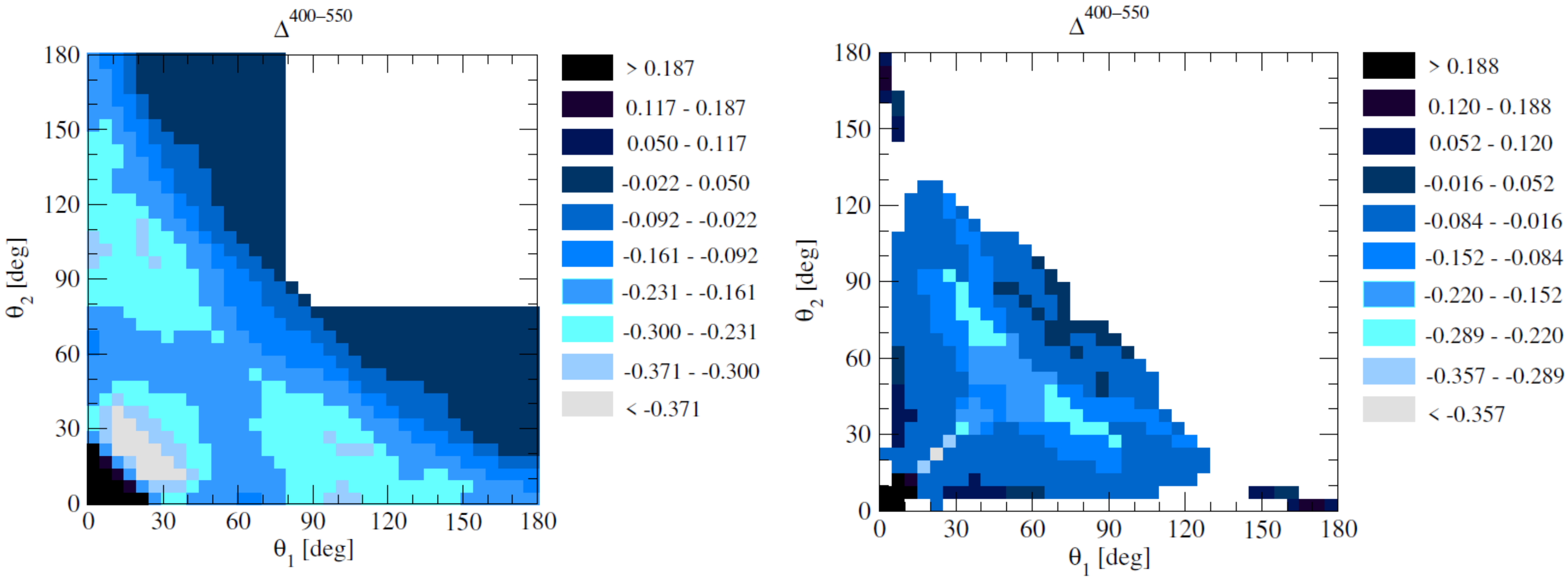
1. Dependence of predictions on the regulator parameter Λ

$$\delta^{400-550}(\theta_1, \theta_2, \phi_2, S) \equiv \frac{\left(\frac{d^5\sigma}{d\Omega_1 d\Omega_2 dS}\right)^{400} - \left(\frac{d^5\sigma}{d\Omega_1 d\Omega_2 dS}\right)^{550}}{\frac{1}{2}\left(\left(\frac{d^5\sigma}{d\Omega_1 d\Omega_2 dS}\right)^{400} + \left(\frac{d^5\sigma}{d\Omega_1 d\Omega_2 dS}\right)^{550}\right)}$$

For each (θ_1, θ_2) we look for maximum of $\delta^{400\text{-}550}$ over Φ_{12} and S(=E_1,E_2)

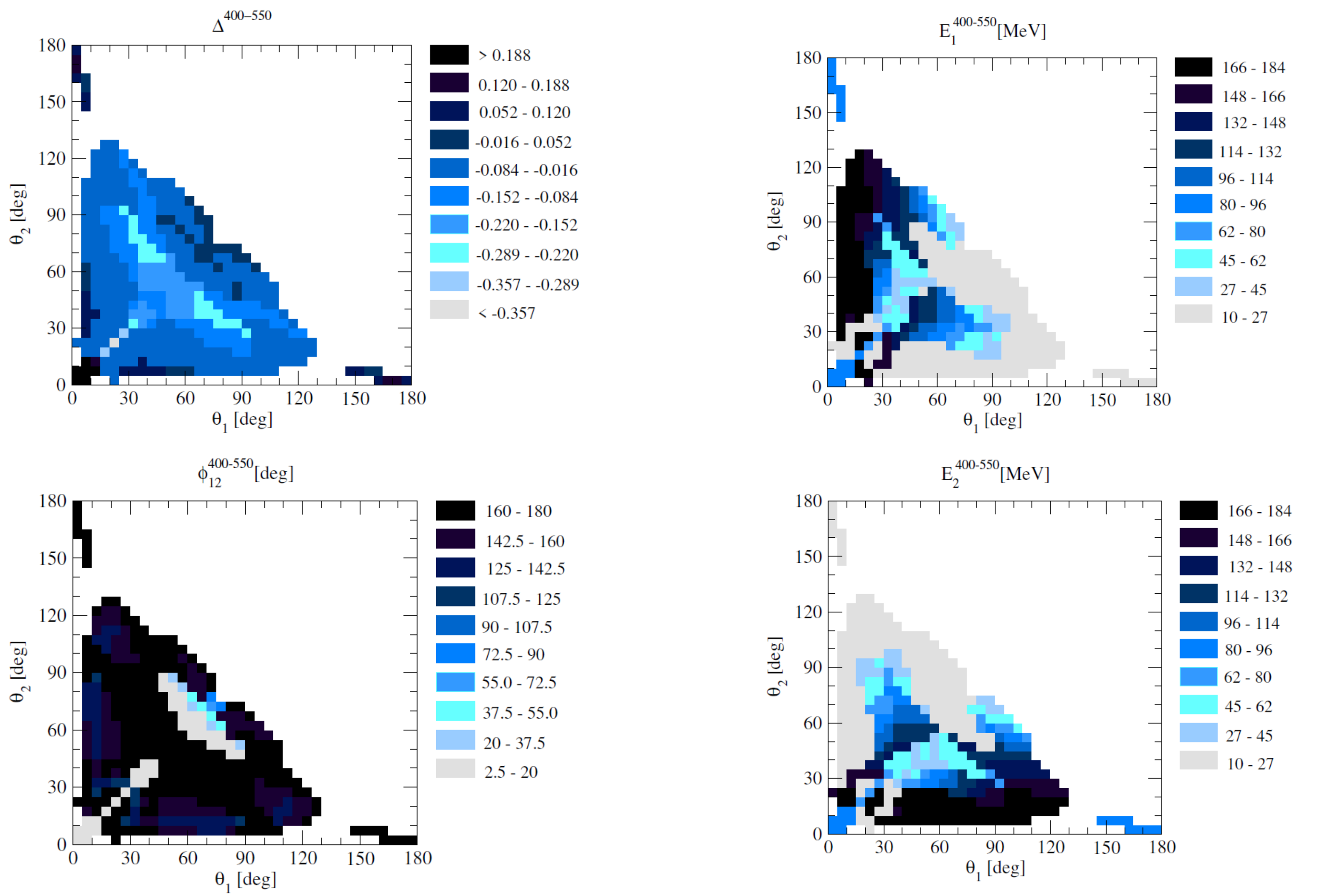
$$\Delta^{400-550} \equiv \Delta^{400-550}(\theta_1, \theta_2) \equiv \max_{\{\phi_2, S\}} \delta^{400-550}(\theta_1, \theta_2, \phi_2, S).$$

Results: E=200 MeV regulator dependence: N4LO+ +N2LO at Λ=400 MeV vs Λ=550 MeV

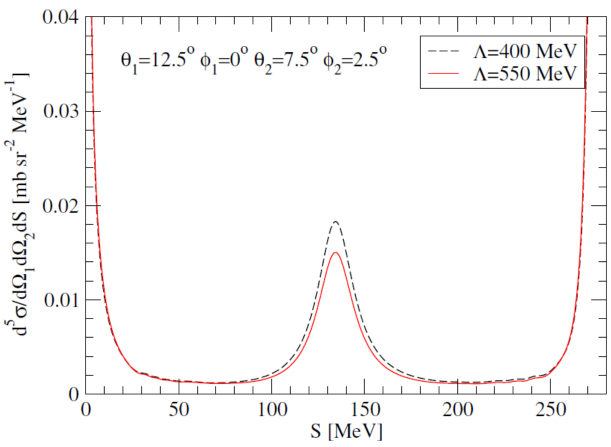


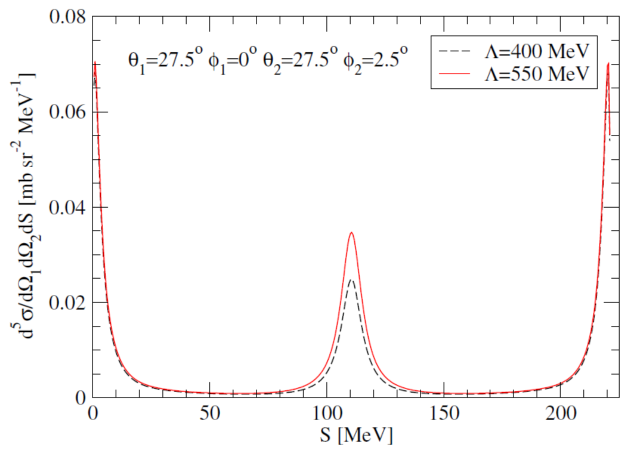
- Right: with additional thresholds:
  $d^5\sigma/d\Omega_1 d\Omega_2 dS \geq 0.01$mb, $E_1 \geq 10$MeV, $E_2 \geq 10$MeV

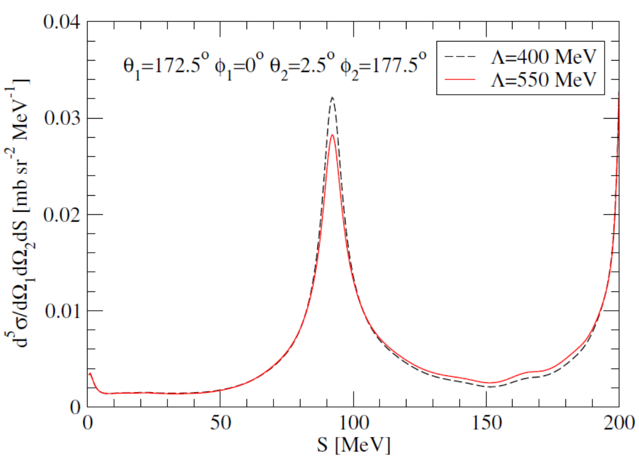
Results: E=200 MeV regulator dependence: N4LO+ + N2LO at Λ=400 MeV vs Λ=550 MeV



Results: E=200 MeV regulator dependence: N4LO+ + N2LO at Λ=400 MeV vs Λ=550 MeV







→ the biggest effects are seen for the FSI configurations

Initiatives on statistical methods

- ISNET (Information and Statistics in Nuclear Experiment and Theory)
  https://isnet-series.github.io/
  The mission of the ISNET community is to encourage, facilitate and develop the use of statistical and computational methodologies that maximize the knowledge gained from and quantify uncertainties in nuclear measurements and theoretical calculations. We do this by combining domain knowledge from the nuclear physics research community with statisticians, mathematicians and computer scientists.

- BAND (Bayesian Analysis of Nuclear Dynamics)
  https://bandframework.github.io/
  The BAND Framework will use advanced statistical methods to produce forecasts for as-yet-unexplored situations that combine nuclear-physics models in an optimal way.

- BUQEYE (Bayesian Uncertainty Quantification: Errors in Your EFT)
  https://buqeye.github.io/
  The BUQEYE Collaboration aims to use statistical tools to answer fundamental problems in the construction and application of effective field theories (EFTs), with particular attention to low-energy nuclear physics. This includes Bayesian parameter estimation, model checking, model selection, and experimental design. We also develop emulators to facilitate these applications.

$\longrightarrow$ Check for experts, knowledge, references and software !

$\longrightarrow$ back to 3NF and its free parameters

$\longrightarrow$ emulators

Fixing parameters of 3NF

- Up to know, i.e. when working at N2LO there are only two free parameters cD and cE.
- Typically, ^3H and the $^2a_{nd}$ or the differential Nd elastic scattering cross section at one or few energies are used.
  The latter requires solving the triton many times and the Faddeev equation 10-20 times.

- However, beyond N2LO we expect:
- No new 3NF free parameters at N3LO, but three new offshell LECs in the chiral NN force.
- 13 contact terms at N4LO (more precisely, due to some identities between operators, one expect in total 13 free parameters of 3NF at N4LO).
- Thus, finding an efficient emulator for solving the 3N Faddeev equation seems to be essential and of the high priority.

Emulators

Various emulators have been proposed recently:

- A.Gnech, et al. arXiv:2511.01844 [nucl-th], arXiv:2511.10420 [nucl-th] p-d scattering at low energies, combines also machine learning, use reduced basis method.
- J.M.Munoz, et al., arXiv:2603.26905 [nucl-th] FRAME emulator, up to ^{55}Ca, combines also machine learning.
- X. Zhang and R. J. Furnstahl, Phys. Rev. C 105, (2022) 064004.
- H.Witała et al., Few-Body Syst. 62 (2021) 23, Eur. Phys. J. A57 (2021) 241, Phys. Rev. C105 (2022) 054004.
- S.Heihoff *in preparation*

- The computational speed-up is several orders of magnitude maintaining the necessary precision.
- New uncertainty is introduced – fortunately, it is small and well under control.

- See also review: T.Duguet, A.Ekström, R.J.Furnstahl, S.König, D.Lee, Rev. Mod. Phys. 96, 031002 (2024)

Emulators – example of application

- Specifically, in H.Witała et al., Phys. Rev. C105 (2022) 054004
  we used the SMS N4LO+ NN potential in combination with the N2LO chiral 3NF supplemented by all the N4LO contact terms. Our aim was to verify if it would be possible to fix strengths of all the contact terms by performing a least squares fit of theory to Nd elastic-scattering data.

- We used SMS N4LO+ NN potential at Λ=450 MeV, combined with the N2LO chiral 3NF and supplemented by all subleading N4LO 3NF contact terms from:
  1. L. Girlanda, A. Kievsky, and M. Viviani, Phys. Rev. C 84, 014001 (2011).,
  2. L. Girlanda, A. Kievsky, and M. Viviani, Phys. Rev. C 102, 019903(E) (2020).

- Relevant Hamiltonian comprises altogether 15 short-range contributions to 3NF, two from N2LO with the strengths cD and cE, and thirteen from N4LO with the strengths E_i, i = 1, . . . , 13. However, for two pairs of the E_i terms matrix elements are identical, thus finally there are 13 unknown parameters.

Emulator for Nd scattering - algorithm

- The contact terms are restricted to small 3N total angular momenta and to only few partial-wave states for a given total 3N angular momentum J and parity π

- Let us spilt 3NF

$$\theta = \{c_1, c_2, \ldots, c_n\}$$

$$\theta_0 = \{0, 0, \ldots, 0\}$$

$$V_{123}^{(1)} = V(\theta_0) + \Delta V(\theta) \equiv V(\theta_0) + \sum_{i=1}^{n} c_i \Delta V_i$$

- We divide the 3N partial-wave states into two sets:

1. The β set is defined by non-vanishing matrix elements of parameters dependent short-range 3NF: ΔV(θ).
2. The α set comprises remaining states.

- Similarly to 3NF

$$T = T(\theta_0) + \Delta T(\theta)$$

Emulator for Nd scattering - algorithm

- Inserting this to the Faddeev equation leads to sets of equations:

  one Eq. is the standard Faddeev equation with $V(\theta_0)$, the second one is for the $\langle\beta|\Delta T(\theta)|\phi\rangle$
- We neglect term $\sim\Delta V\Delta T$
- We split that Eq. to separate equations, each for single parameter dependent component of V: $V_i=c_iV$. We may solve that equation separately at $c_i=1$ obtaining corresponding ΔT_i .
- Finally:

$$\langle\beta|\Delta T(\theta)|\phi\rangle = \sum_{i=1}^{N} c_i\langle\beta|\Delta T_i|\phi\rangle.$$

- Summarizing: one needs to solve N+1 Faddeev equations (one for $T(\theta_0)$) and N for $\langle\beta|\Delta T_i|\phi\rangle$), next N times find $\langle\alpha|\Delta T_i|\phi\rangle$ by integration.

Emulator for Nd scattering - algorithm

- In this way we have matrix elements of T

$$\langle \alpha | T(\theta) | \phi \rangle = \langle \alpha | T(\theta_0) | \phi \rangle + \sum_i c_i \langle \alpha | \Delta T_i | \phi \rangle,$$

$$\langle \beta | T(\theta) | \phi \rangle = \langle \beta | T(\theta_0) | \phi \rangle + \sum_i c_i \langle \beta | \Delta T_i | \phi \rangle. \quad (10)$$

- Let us now come back to the scattering amplitudes

$$U = PG_0^{-1} + V_{123}^{(1)}(1+P)\phi + PT + V_{123}^{(1)}(1+P)G_0 T$$

$$U_0 = (1+P)T$$

- They are linear in T: the dependence on the c_i constants carries over to them, except as a complication for elastic scattering, but they can be written as

$$U = U(\theta_0) + \sum_i c_i U_i + \sum_{i,k} c_i c_k U_{ik}$$

$$U_0 = U_0(\theta_0) + \sum_i c_i U_{0i}$$

Emulator for Nd scattering – fit to the true data at 10,70, and 135 MeV (786 data points)

TABLE III. The values of strengths c_i found in the least squares fit to the data from Table II at the three energies $E = 10$, 70, and 135 MeV.

c_D	-1.49 ± 0.06
c_E	-1.27 ± 0.06
c_{E_1}	6.40 ± 0.33
c_{E_2}	7.80 ± 0.36
c_{E_3}	6.97 ± 0.34
c_{E_4}	-2.06 ± 0.13
c_{E_5}	-0.36 ± 0.05
c_{E_6}	0.52 ± 0.03
c_{E_7}	-7.40 ± 0.14
c_{E_8}	-2.61 ± 0.05
c_{E_9}	-4.59 ± 0.22
$c_{E_{10}}$	-0.98 ± 0.05
$c_{E_{13}}$	-1.14 ± 0.05

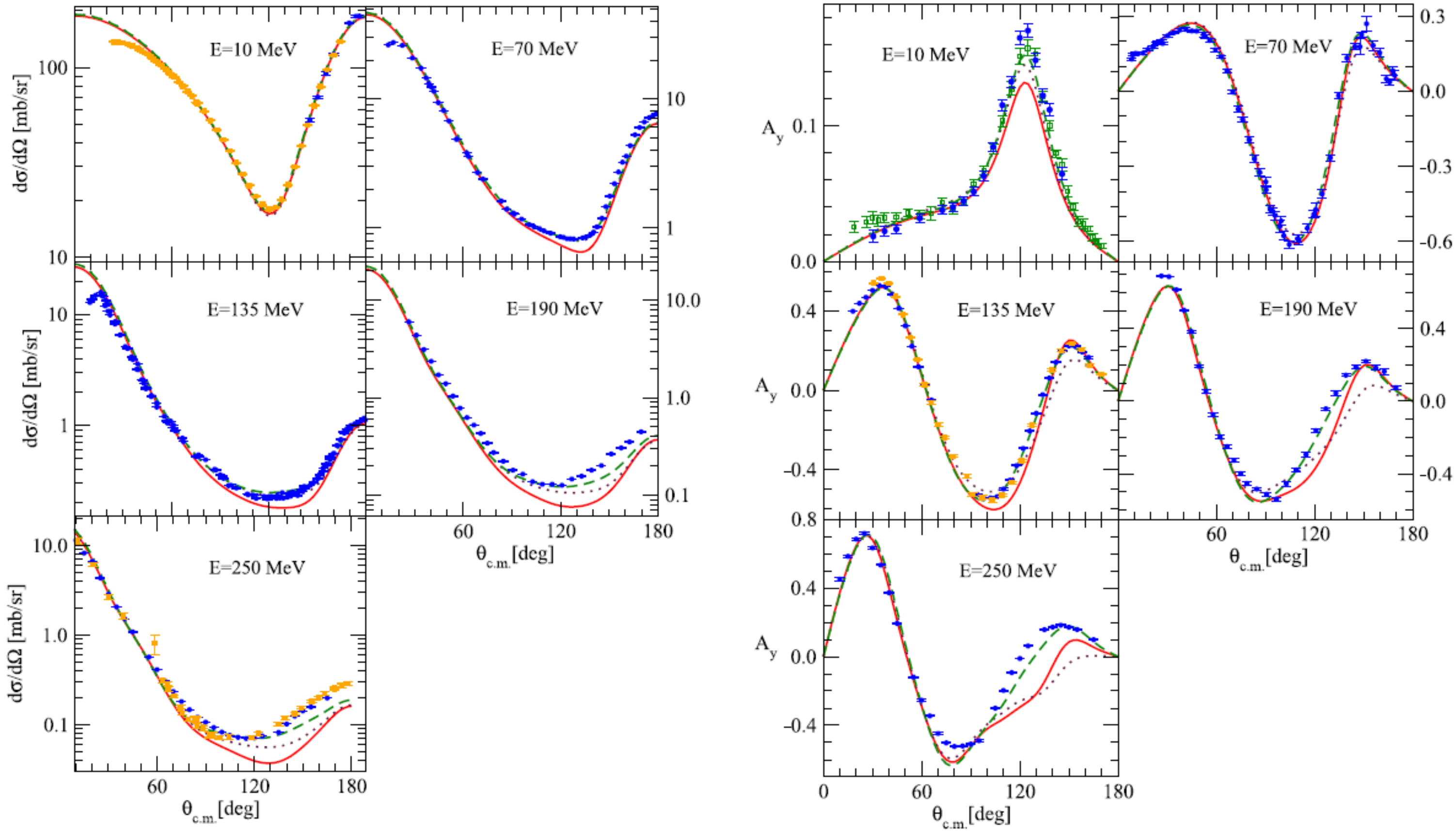
TABLE IV. The covariance matrix for the strengths c_i determined by the least squares fit of data from Table II at the three energies $E = 10$, 70, and 135 MeV [the values shown are $\mathrm{Cov}(c_i, c_j) \times 1000$].

	c_D	c_E	c_{E_1}	c_{E_2}	c_{E_3}	c_{E_4}	c_{E_5}	c_{E_6}	c_{E_7}	c_{E_8}	c_{E_9}	$c_{E_{10}}$	$c_{E_{13}}$
c_D	3.914	−0.456	1.412	4.573	0.843	0.844	−0.729	−0.892	1.109	0.267	−0.726	0.123	−0.207
c_E		3.560	0.947	−3.571	1.345	−0.633	−0.172	−0.217	−2.416	−0.809	−1.702	0.393	0.571
c_{E_1}			108.9	112.8	108.9	−35.13	1.409	−2.418	25.92	7.513	12.99	3.861	0.443
c_{E_2}				130.7	113.4	−35.15	−1.995	−3.241	32.43	9.561	−0.534	0.763	−3.332
c_{E_3}					112.9	−38.92	1.617	−1.814	27.52	8.068	8.366	1.598	−0.193
c_{E_4}						15.97	−1.966	−0.362	−10.50	−3.198	−4.866	0.345	−0.222
c_{E_5}							2.415	0.669	0.791	0.281	9.892	1.311	1.766
c_{E_6}								0.635	−0.874	−0.226	1.426	−0.226	0.210
c_{E_7}									20.33	6.455	3.464	−0.324	−1.463
c_{E_8}										2.071	1.041	−0.158	−0.462
c_{E_9}											50.23	9.133	8.813
$c_{E_{10}}$												2.625	1.910
$c_{E_{13}}$													2.499

- Big values of $c_{E1}, c_{E2}, c_{E3}, c_{E7}, c_{E9}$
- Correlation coefficients close to ± 1: $\rho(E_1,E_2)$, $\rho(E_2,E_3)$, $\rho(E_1,E_3)$, $\rho(E_3,E_4)$, $\rho(E_7,E_8)$
- Correlation coefficients close to 0: (c_D,c_E),(c_D,c_{Ei}),(c_E,c_{Ei})
- χ^2/data≈35

Emulator for Nd scattering – fit to the data: cross section and $A_Y(N)$

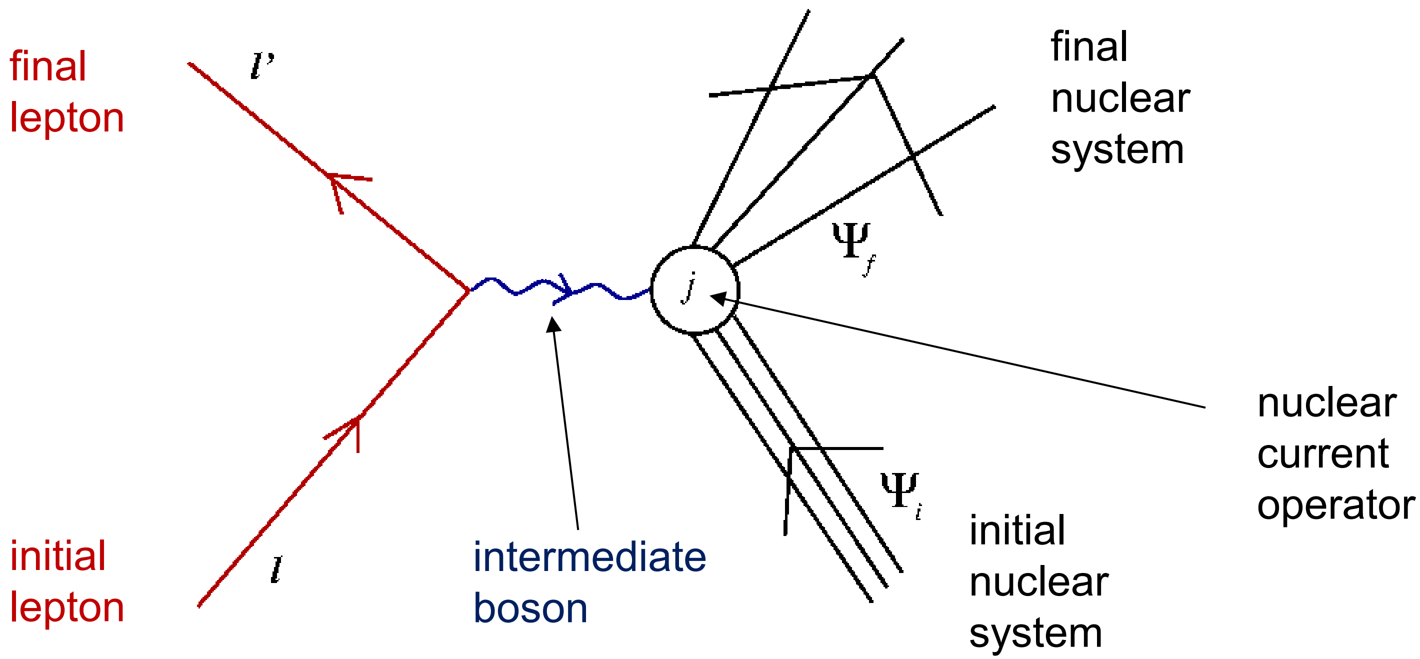
- Data at 10, 70 and 135 MeV
- Results at 190 and 250 MeV are predictions



NN N4LO+ NN N4LO+ + 3NF N2LO NN N4LO+ + 3NF N2LO + E_i

Beyond pure 2N and 3N systems - electroweak probes

Formalism



$$\left|T_{fi}\right|^2 \propto L_{\alpha\beta}\, N^{\alpha}\left(N^{\beta}\right)^{*} \qquad N^{\alpha} \equiv \left\langle \Psi_f \left| j^{\alpha} \right| \Psi_i \right\rangle$$

$L_{\alpha\beta}$ known analytically ! (spinors and gamma matrices)

Formalism

$N^{\alpha} = \left\langle \Psi_{f m_f} \left| j^{\alpha} \right| \Psi_{i m_i} \right\rangle$ from *ab initio* calculations in momentum space

Dynamical ingredients:

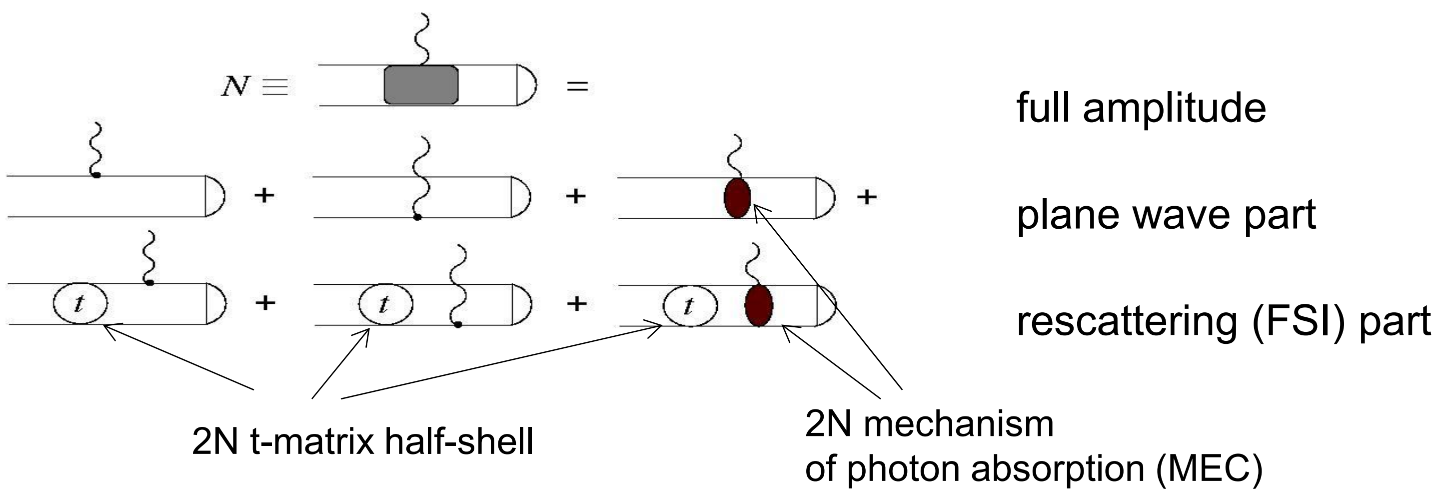
(1): 2N and 3N Hamiltonians $H_{2N} = H_0^{2N} + V_{12}$

$$H_{3N} = H_0^{3N} + V_{23} + V_{13} + V_{12} + V_{123}$$

used to generate nuclear bound and scattering states contain 2N and 3N potentials.

(2): nuclear EM and weak single-nucleon, 2N current operators, and many-body currents for A>2

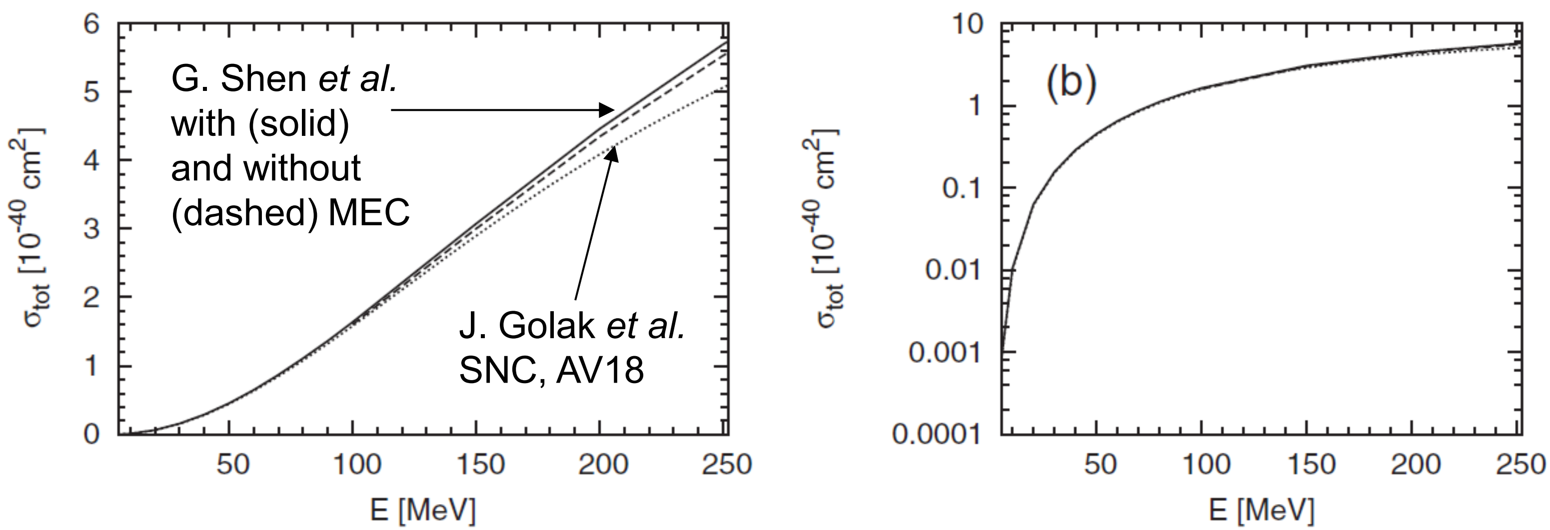
$j_{2N} = j_1 + j_2 + j_{12}$ describe interactions of the electroweak probe with nuclear system.



Reactions with (anti)neutrinos

$$\bar{\nu}_e + {}^2H \rightarrow e^+ + n + n$$

Total CC cross section as a function of the antineutrino energy

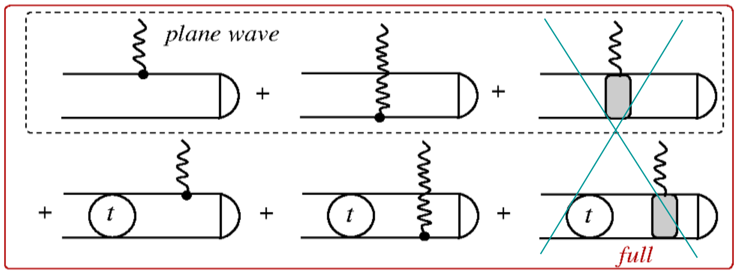


Comparison with G. Shen *et al.*, Phys. Rev. C 86, 035503 (2012): difference between dotted and dashed curves is due relativistic kinematics for nucleons.

Extension to relativistic regime

Inelastic electron scattering off the deuteron: $e + d \rightarrow e' + p + n$

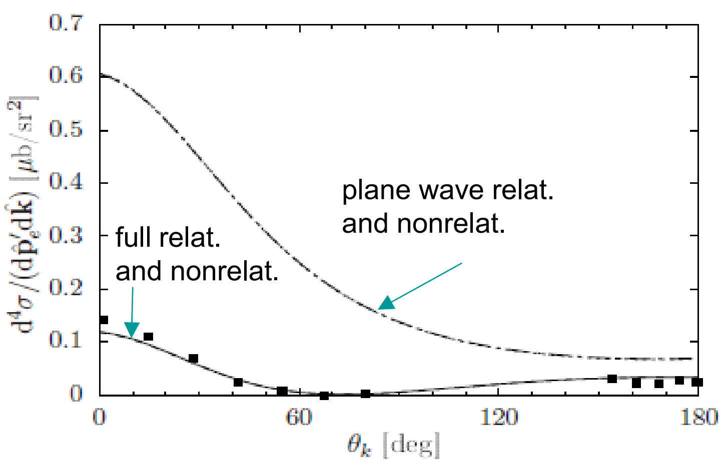
Exclusive cross section: $\dfrac{d^5\sigma}{d\hat{\mathbf{p}}'_e d|\mathbf{p}'_e| d\hat{\mathbf{p}}'_1}$



For nucleon

$$\langle \mathbf{p}'_1 \mu'_1 \tau'_1 \mathbf{p}'_2 \mu'_2 \tau'_2 | j(1) | \phi_d \mu_d \mathbf{P}_i \rangle$$

$$\langle \mathbf{p}'_1 \mu'_1 \tau'_1 \mathbf{p}'_2 \mu'_2 \tau'_2 | t\, G_0\, j(1) | \phi_d \mu_d \mathbf{P}_i \rangle$$



Exp: P. Von Neumann-Cosel et al., PRL 88 (2002) 202304

$$\frac{d^4\sigma}{d\hat{p}'_e d\hat{k}} = \int_{E_e'^{\min}}^{E_e'^{\max}} dE'_e \frac{d^5\sigma}{dE'_e d\hat{p}'_e d\hat{k}}$$

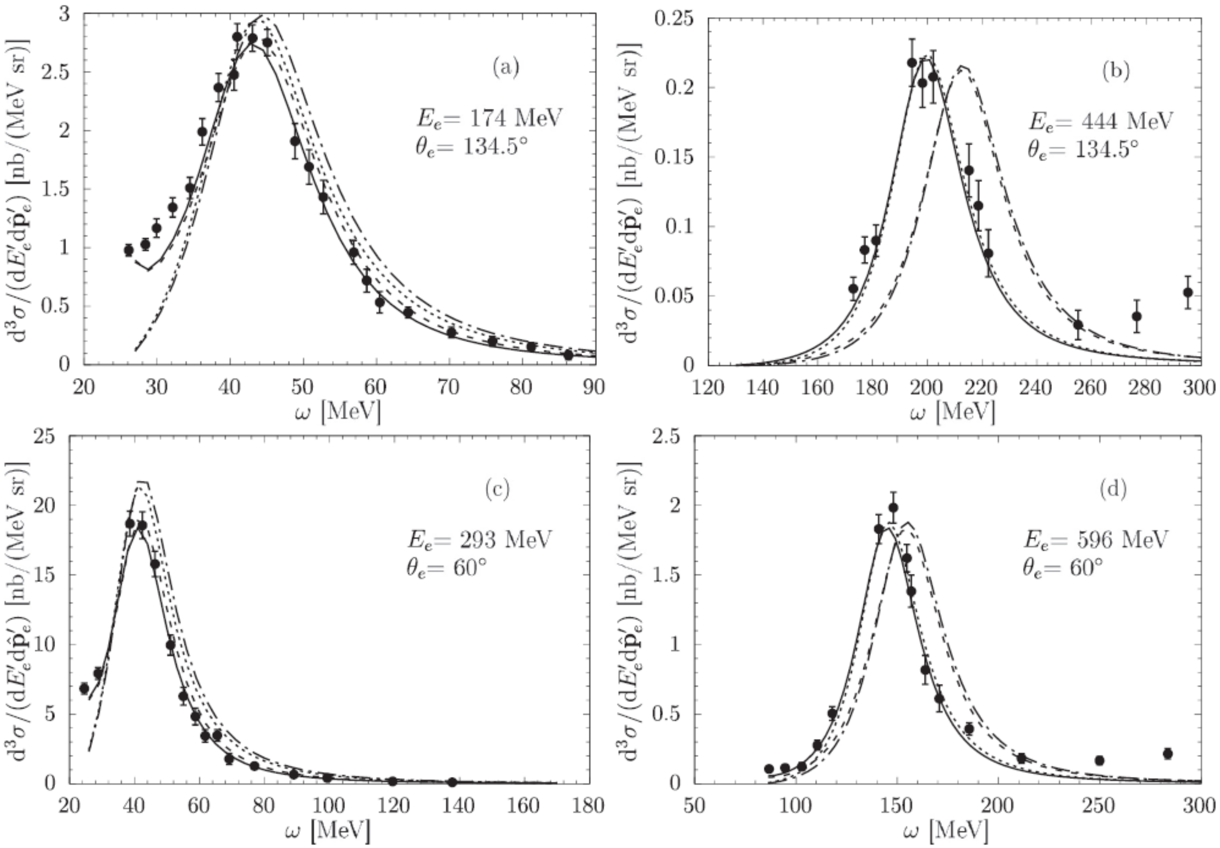
E=85 MeV, θ_e=40°

Rescattering part is important (at least in that kinematics).

Extension to relativistic regime

Semi-inclusive cross section for the ^{2}H(e,e') reaction $\frac{d^3\sigma}{dE'_e\, d\hat{p}'_e}$ as a function of energy transfer ω (only electron in the final state is detected)

Exp: B.P.Quinn et al., Phys. Rev. C37 (1988) 1609



Nonrel PW
Nonrel FULL
Relat PW
Relat FULL

→rescattering part is important at low energy transfer

→at high energy transfer some dynamic is missing

More:
A.Grassi et al. Phys. Rev. C107 (2023) 024617

Kharkiv potential

I. Dubovyk · O. Shebeko, Few-Body Syst (2010) 48: 109
A.V.Shebeko, M.I.Shirokov, Phys.Part.Nucl. 32 (2001) 15
Greenberg, O.W., Schweber, S.S. *Nuovo Cim* **8**, 378 (1958)

Lagrangian for nucleons and boson fields

Applying on it the Unitary Clothing Transformations (UCT) to express the original field Hamiltonian through new creation and annihilation operators of the clothed mesons and nucleons.
The transformation allows for accumulation of virtual processes induced with the meson absorption/emission, the *N-\bar(N)* –pair annihilation and production and other cloud effects in the interactions between the clothed particles, responsible for different physical (not virtual) processes in the NN scattering).

Mesons: π–, η–, ρ–, ω–, δ– and σ-mesons, like Bonn potential
Arising interaction is energy independent, fitted to 2N phase shifts

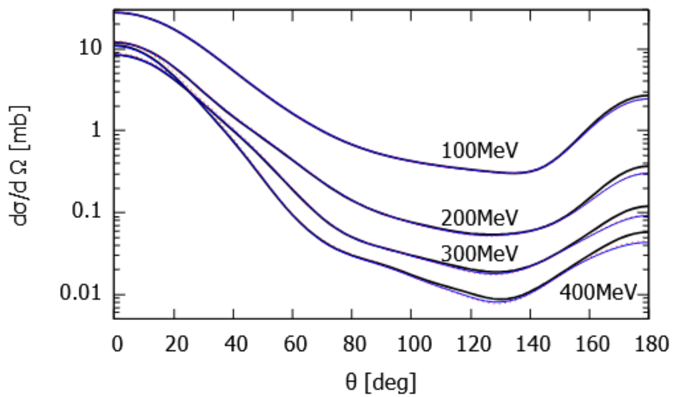
^{3}H:

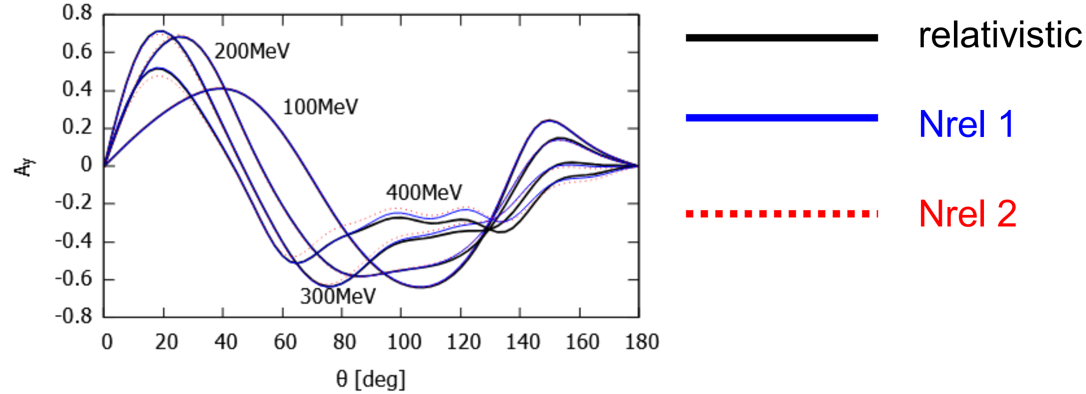
Table 1 The theoretical predictions of the triton binding energy resulting from the solutions of nonrelativistic (first row) and relativistic (second row) Faddeev equations

	CD-Bonn	AV18	Nijmegen II	Nijmegen I	Nijmegen 93	Kharkov
Nonrelativistic cal.	−8.33	−7.66	−7.65	−8.00	−7.76	(−7.49)
Relativistic cal.	(−8.22)	(−7.59)	(−7.58)	(−7.90)	(−7.68)	−7.42
Difference (rel.–nonrel.)	0.11	0.07	0.07	0.10	0.08	0.07

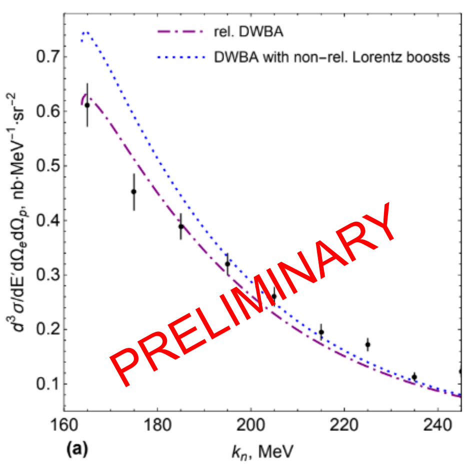
Kharkiv potential

- Relativistic effects in Nd scattering (H.Kamada et al., arXiv:2601.00534 [nucl-th])





- Deuteron electrodisintegration



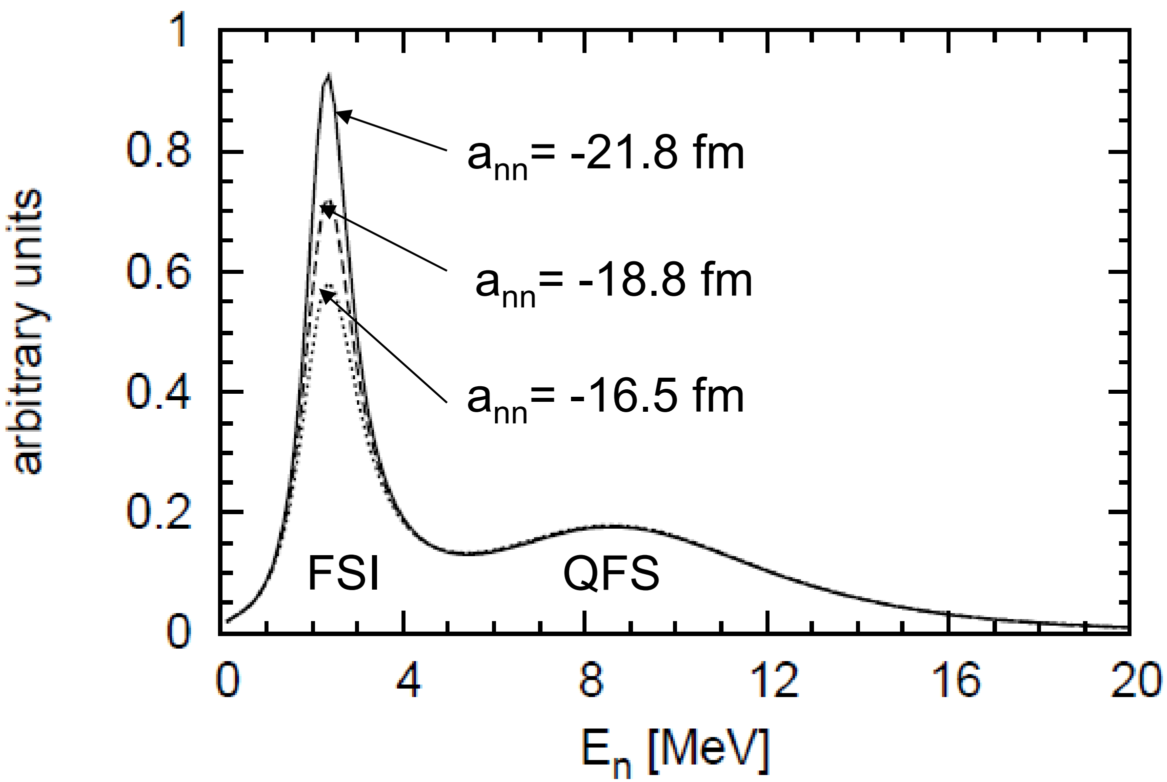
O.Shebeko, **A.Arslanaliev, Y.Kostylenko**,M.Brady, V.Chahar, **J.Golak**, H.Kamada, W.N.Polyzou, D.Ramirez, R.Skibiński, K.Topolnicki, and H.Witała,
in preparation:

Supported by the National Science Centre, Poland, under Grant No. IMPRESS-U 2024/06/Y/ST2/00135

Radiative pion capture on ^{2}H

$$\pi^- + d \to \gamma + n + n$$

$$d\Gamma^5_{nn}/(d\Omega_\gamma\, d\Omega_1\, dE_1)$$

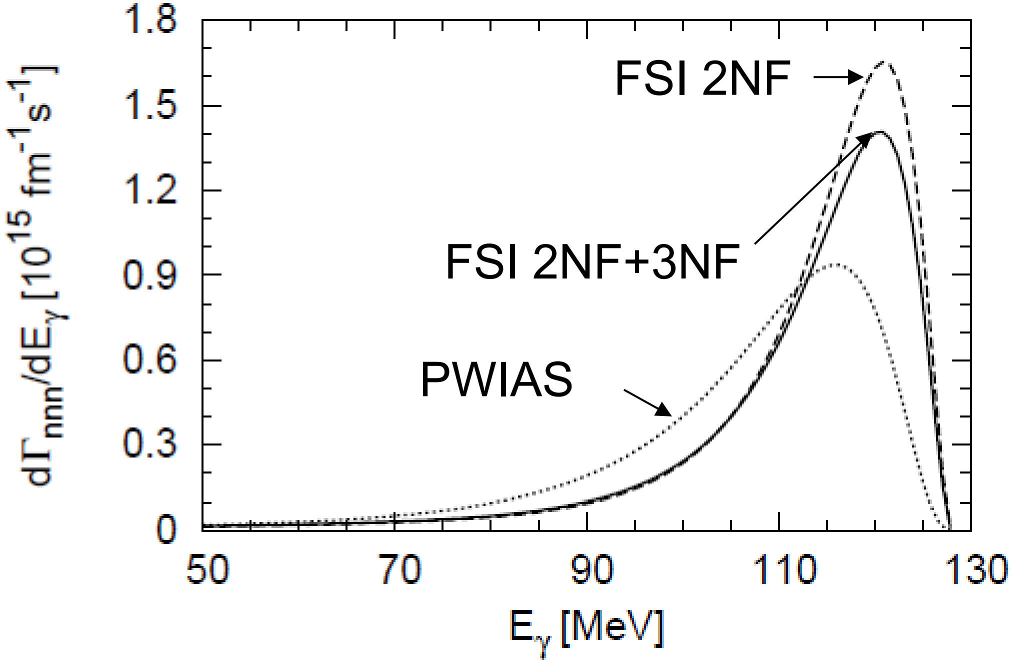


neutron-neutron potential is changed by 1 % only in the $^1S_0$ channel

neutron energy spectra for $\theta_{\gamma 1} = 179^\circ$

Radiative pion capture on ^{3}H: three-neutron breakup

$$\pi^- + {}^3\mathrm{H} \to \gamma + n + n + n$$



$\Gamma_{nnn} = 0.117 \times 10^{15}$ 1/s (PWIAS)
$\Gamma_{nnn} = 0.141 \times 10^{15}$ 1/s (FSI 2NF)
$\Gamma_{nnn} = 0.128 \times 10^{15}$ 1/s (FSI 2NF+3NF)

→ both FSI and 3NF are important

More: J.Golak at al., Phys.Rev. C98 (2018) 054001

Search for quantum entanglement in nuclear systems

Entanglement in final polarization states of the neutron-proton scattering: entangled state

- By entanglement we mean strong (maximal) correlation between results of measurement of neutron and proton spin projections

- Bell states

$$
\begin{aligned}
|\psi_I\rangle &= \frac{1}{\sqrt{2}}(|+\frac{1}{2}+\frac{1}{2}\rangle + |-\frac{1}{2}-\frac{1}{2}\rangle) \\
|\psi_{II}\rangle &= \frac{1}{\sqrt{2}}(|+\frac{1}{2}+\frac{1}{2}\rangle - |-\frac{1}{2}-\frac{1}{2}\rangle) \\
|\psi_{III}\rangle &= \frac{1}{\sqrt{2}}(|-\frac{1}{2}+\frac{1}{2}\rangle + |+\frac{1}{2}-\frac{1}{2}\rangle) \\
|\psi_{IV}\rangle &= \frac{1}{\sqrt{2}}(|+\frac{1}{2}-\frac{1}{2}\rangle - |-\frac{1}{2}+\frac{1}{2}\rangle)
\end{aligned}
$$

- For Bell states the spin correlation $\langle \sigma_i^n \sigma_i^p \rangle = \pm 1$, $\left\langle \sigma_i^n \sigma_{\mathrm{j}}^p \right\rangle = 0$ and $\left\langle \sigma_i^{n(p)} \right\rangle = 0$.

Entanglement in final polarization states of the neutron-proton scattering: entangled state

We will check:

1) If spin correlations are close to -1, 0 or +1

   $\langle \sigma_x^n \sigma_x^p \rangle$, $\langle \sigma_y^n \sigma_y^p \rangle$, $\langle \sigma_z^n \sigma_z^p \rangle$, $\langle \sigma_x^n \sigma_z^p \rangle$, $\langle \sigma_z^n \sigma_x^p \rangle$

2) If nucleon's polarization $\left\langle \sigma_y^{n(p)} \right\rangle$ close to 0

3) Idempotent condition for density matrix

Assumptions: proton and neutron in initial state prepared separately (no spin correlation), both have polarization only in y-direction.

Using LS eq. we compute (with AV18 force) the induced neutron-proton spin correlations, induced nucleons polarizations and spin correlation transfers and double spin correlation transfers.

Entanglement in final polarization states of the neutron-proton scattering: polarized initial state $\sigma_y^n, \sigma_y^p = \pm 1$

Entanglement power $\epsilon(\rho_{np}) = 1 - \mathrm{Tr}\left(\rho_n^2\right)$

Concurrence

$$C(\rho_{np}) \equiv \frac{1}{2} \mid < \hat{\sigma}_y^n \otimes \hat{\sigma}_y^p > \mid = \frac{1}{2} \left|\mathrm{Tr}\left(\rho_{np}\, \hat{\sigma}_y^n \otimes \hat{\sigma}_y^p\right)\right|$$

$$\bar{C}(\rho_{np}) \equiv \frac{1}{2} \sqrt{1 - \langle\hat{\sigma}_x^n\rangle^2 - \langle\hat{\sigma}_y^n\rangle^2 - \langle\hat{\sigma}_y^n\rangle^2} = \frac{1}{2} \sqrt{1 - \langle\hat{\sigma}_y^n\rangle^2}$$

Equals 0.5 for Bell states

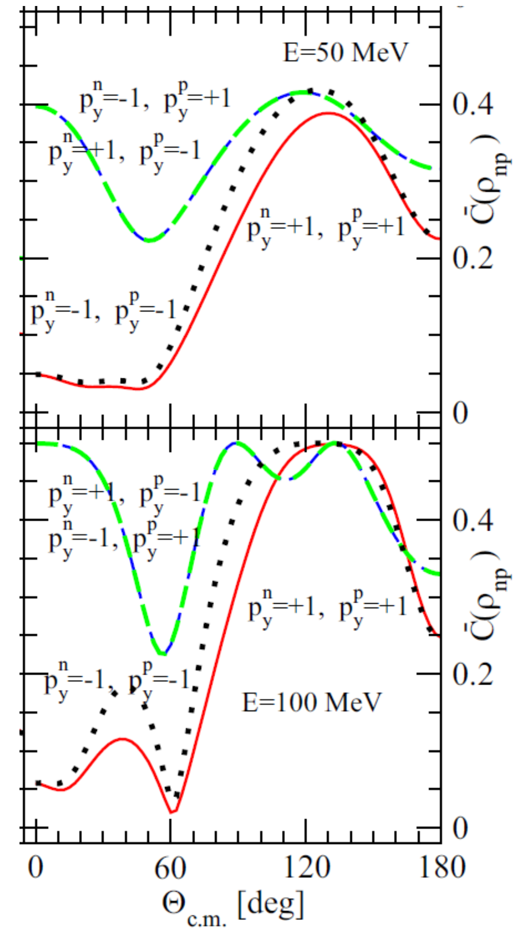
- Angular dependence

→ find maximum

→ find polarizations and spin correlations

→ find α coefficients

→ compare with the Bell states



Entanglement in final polarization states of the neutron-proton scattering: polarized initial state $\sigma_y^n, \sigma_y^p = \pm 1$

- Search over the scattering angle reveals specific states (Tab. 3, H.Witała et al. Phys Rev C112 (2025) 044002)

$E^{a)}$ (MeV)	$\Theta_{c.m.}$ (deg)	$\alpha_{+\frac{1}{2}+\frac{1}{2}}$	$\alpha_{+\frac{1}{2}-\frac{1}{2}}$	$\alpha_{-\frac{1}{2}+\frac{1}{2}}$	$\alpha_{-\frac{1}{2}-\frac{1}{2}}$	$\lvert\alpha_{+\frac{1}{2}+\frac{1}{2}}\rvert^2$ $\lvert\alpha_{-\frac{1}{2}-\frac{1}{2}}\rvert^2$	$\lvert\alpha_{+\frac{1}{2}-\frac{1}{2}}\rvert^2$ $\lvert\alpha_{-\frac{1}{2}+\frac{1}{2}}\rvert^2$	Bell type
50 [(a)]	106.5	$-0.268 - i0.242$	$-0.584 + i0.167$	$-0.584 + i0.167$	$+0.268 + i0.242$	0.131	0.370	$\lvert\psi_{III}\rangle$
50 [(b)]	0.0	$-0.378 - i0.517$	$-0.300 - i0.008$	$+0.300 + i0.008$	$-0.378 - i0.517$	0.410	0.090	$\lvert\psi_I\rangle$
50 [(b)]	-					0.175	0.325	
50 [(c)]	0.0	$-0.378 - i0.517$	$+0.300 + i0.008$	$-0.300 - i0.008$	$-0.378 - i0.517$	0.410	0.090	$\lvert\psi_I\rangle$
50 [(c)]	-					0.175	0.325	
50 [(d)]	123.5	$+0.379 + i0.561$	$+0.175 - i0.103$	$+0.175 - i0.103$	$-0.379 - i0.561$	0.459	0.041	$\lvert\psi_{II}\rangle$

$$|\psi_{II}\rangle = \frac{1}{\sqrt{2}}(|+\frac{1}{2}+\frac{1}{2}\rangle - |-\frac{1}{2}-\frac{1}{2}\rangle)$$

- (d) states for inital polarizations (-1, -1)

- Best finding (E=100 MeV)

100 [(d)]	143.0	$+0.602 + i0.360$	$-0.012 - i0.091$	$-0.012 - i0.091$	$-0.602 - i0.360$	0.492	0.009	$\lvert\psi_{II}\rangle$

Entanglement in the neutron-deuteron scattering

- Elastic scattering

No	E [(x)] MeV	$\Theta_{c.m.}$ deg	$\bar{\alpha}_{+\frac{1}{2}+1}$ $\bar{\alpha}_{-\frac{1}{2}-1}$	$\bar{\alpha}_{+\frac{1}{2}0}$ $\bar{\alpha}_{-\frac{1}{2}0}$	$\bar{\alpha}_{+\frac{1}{2}-1}$ $\bar{\alpha}_{-\frac{1}{2}+1}$	$\lvert\bar{\alpha}_{++}\rvert^2$ $\lvert\bar{\alpha}_{--}\rvert^2$	$\lvert\bar{\alpha}_{+0}\rvert^2$ $\lvert\bar{\alpha}_{-0}\rvert^2$	$\lvert\bar{\alpha}_{+-}\rvert^2$ $\lvert\bar{\alpha}_{-+}\rvert^2$	Bell type	A	B	C	D
8	135 [(a)]	88	$-0.063 + i0.113$ $+0.113 + i0.063$	$+0.166 - i0.006$ $+0.006 + i0.166$	$+0.542 + i0.403$ $+0.403 - i0.542$	0.017	0.027	0.456	$\lvert\psi_{II}^{nd}\rangle$	+0.505	+1.495	+0.010	+0.147
					should be	0	0	0.5		0.5	1.5	0	0

- Deuteron breakup (only some FSI and QFS kinematics and specific initial polarizations are investigated, entanglement for N1,N2).
- For QFS and FSI(np) states close to the Bell states but with small entanglement-spoiling component.

No	E [(a)] MeV	θ_1^{lab} deg	$\bar{\alpha}_{+++}$ $\bar{\alpha}_{---}$	$\bar{\alpha}_{++-}$ $\bar{\alpha}_{--+}$	$\bar{\alpha}_{+-+}$ $\bar{\alpha}_{-+-}$	$\bar{\alpha}_{+--}$ $\bar{\alpha}_{-++}$	$\lvert\bar{\alpha}_{+++}\rvert^2$ $\lvert\bar{\alpha}_{---}\rvert^2$	$\lvert\bar{\alpha}_{++-}\rvert^2$ $\lvert\bar{\alpha}_{--+}\rvert^2$	$\lvert\bar{\alpha}_{+-+}\rvert^2$ $\lvert\bar{\alpha}_{-+-}\rvert^2$	$\lvert\bar{\alpha}_{+--}\rvert^2$ $\lvert\bar{\alpha}_{-++}\rvert^2$
8	135.0 [(b)]	44.6	$-0.006 - i0.009$ $-0.009 + i0.006$	$+0.019 - i0.017$ $+0.017 + i0.019$	$-0.418 - i0.286$ $+0.286 - i0.418$	$-0.275 + i0.409$ $+0.409 + i0.275$	0.000	0.001	0.256	0.243
				should be			0	0	0.25	0.25

- For FSI(nn) there are pure and maximally entanglement states.

More details: H.Witała, Eur. Phys. J A62 (2026) 24

entanglement in 2H: Ashutosh Singh, Ankit Kumar Das, Ankit Kumar, P. Arumugam arXiv:2506.16621 [nucl-th]

Entanglement in the neutron-deuteron scattering

- Elastic scattering

No	E [(x)] MeV	$\Theta_{c.m.}$ deg	$\bar{\alpha}_{+\frac{1}{2}+1}$ $\bar{\alpha}_{-\frac{1}{2}-1}$	$\bar{\alpha}_{+\frac{1}{2}0}$ $\bar{\alpha}_{-\frac{1}{2}0}$	$\bar{\alpha}_{+\frac{1}{2}-1}$ $\bar{\alpha}_{-\frac{1}{2}+1}$	$\lvert\bar{\alpha}_{++}\rvert^2$ $\lvert\bar{\alpha}_{--}\rvert^2$	$\lvert\bar{\alpha}_{+0}\rvert^2$ $\lvert\bar{\alpha}_{-0}\rvert^2$	$\lvert\bar{\alpha}_{+-}\rvert^2$ $\lvert\bar{\alpha}_{-+}\rvert^2$	Bell type	A	B	C	D
8	135 [(a)]	88	$-0.063 + i0.113$ $+0.113 + i0.063$	$+0.166 - i0.006$ $+0.006 + i0.166$	$+0.542 + i0.403$ $+0.403 - i0.542$	0.017	0.027	0.456	$\lvert\psi^{nd}_{II}\rangle$	+0.505	+1.495	+0.010	+0.147
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No	E [(a)] MeV	θ_1^{lab} deg	$\bar{\alpha}_{+++}$ $\bar{\alpha}_{---}$	$\bar{\alpha}_{++-}$ $\bar{\alpha}_{--+}$	$\bar{\alpha}_{+-+}$ $\bar{\alpha}_{-+-}$	$\bar{\alpha}_{+--}$ $\bar{\alpha}_{-++}$	$\lvert\bar{\alpha}_{+++}\rvert^2$ $\lvert\bar{\alpha}_{---}\rvert^2$	$\lvert\bar{\alpha}_{++-}\rvert^2$ $\lvert\bar{\alpha}_{--+}\rvert^2$	$\lvert\bar{\alpha}_{+-+}\rvert^2$ $\lvert\bar{\alpha}_{-+-}\rvert^2$	$\lvert\bar{\alpha}_{+--}\rvert^2$ $\lvert\bar{\alpha}_{-++}\rvert^2$
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Entanglement in the neutron-deuteron scattering

- Elastic scattering

No	E [(x)] MeV	$\Theta_{c.m.}$ deg	$\bar{\alpha}_{+\frac{1}{2}+1}$ $\bar{\alpha}_{-\frac{1}{2}-1}$	$\bar{\alpha}_{+\frac{1}{2}0}$ $\bar{\alpha}_{-\frac{1}{2}0}$	$\bar{\alpha}_{+\frac{1}{2}-1}$ $\bar{\alpha}_{-\frac{1}{2}+1}$	$\lvert\bar{\alpha}_{++}\rvert^2$ $\lvert\bar{\alpha}_{--}\rvert^2$	$\lvert\bar{\alpha}_{+0}\rvert^2$ $\lvert\bar{\alpha}_{-0}\rvert^2$	$\lvert\bar{\alpha}_{+-}\rvert^2$ $\lvert\bar{\alpha}_{-+}\rvert^2$	Bell type	A	B	C	D
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No	E [(a)] MeV	θ_1^{lab} deg	$\bar{\alpha}_{+++}$ $\bar{\alpha}_{---}$	$\bar{\alpha}_{++-}$ $\bar{\alpha}_{--+}$	$\bar{\alpha}_{+-+}$ $\bar{\alpha}_{-+-}$	$\bar{\alpha}_{+--}$ $\bar{\alpha}_{-++}$	$\lvert\bar{\alpha}_{+++}\rvert^2$ $\lvert\bar{\alpha}_{---}\rvert^2$	$\lvert\bar{\alpha}_{++-}\rvert^2$ $\lvert\bar{\alpha}_{--+}\rvert^2$	$\lvert\bar{\alpha}_{+-+}\rvert^2$ $\lvert\bar{\alpha}_{-+-}\rvert^2$	$\lvert\bar{\alpha}_{+--}\rvert^2$ $\lvert\bar{\alpha}_{-++}\rvert^2$
8	135.0 [(b)]	44.6	$-0.006-i0.009$ $-0.009+i0.006$	$+0.019-i0.017$ $+0.017+i0.019$	$-0.418-i0.286$ $+0.286-i0.418$	$-0.275+i0.409$ $+0.409+i0.275$	0.000	0.001	0.256	0.243
				should be			0	0	0.25	0.25

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More details: H.Witała, Eur. Phys. J A62 (2026) 24

entanglement in 2H: Ashutosh Singh, Ankit Kumar Das, Ankit Kumar, P. Arumugam arXiv:2506.16621 [nucl-th]

Summary

- A long series of efforts spanning nearly 100 years has resulted in a fairly good understanding of low-energy nuclear physics, but many puzzles and challenges remain.
- There is a hope for further progress in the coming years:
  - completing 3NF at N3LO and at N4LO (?).
  - 2N- electroweak currents, hopefully from χEFT, consistent with 2NF and 3NF.
  - emulators, faster algorithms and availability of supercomputer resources (however here the competition with other disciplines is huge – do the quantum computers solve the problem?)
  - Attacks on the scattering problem beyond A=3N are ongoing (4N, n-4He, nucleon–light nuclei scattering, etc).
  - Nuclear LatticeEFT (D.Lee, U-G.Meiβner, et al.), Low-Resolution EFT, …
- I have presented only part of interesting problems and efforts. But there is much more:
  - Electromagnetic processes (Mainz program), weak processes (DUNE)
  - Effimov physics
  - Tremendous progress in nuclear structure calculations
  - Systems with strangeness
  - ….

THANK YOU!