QUANTUM SHARDULA, MEASUREMENT PROBLEM AND GRAVITATIONAL WAVE DETECTION

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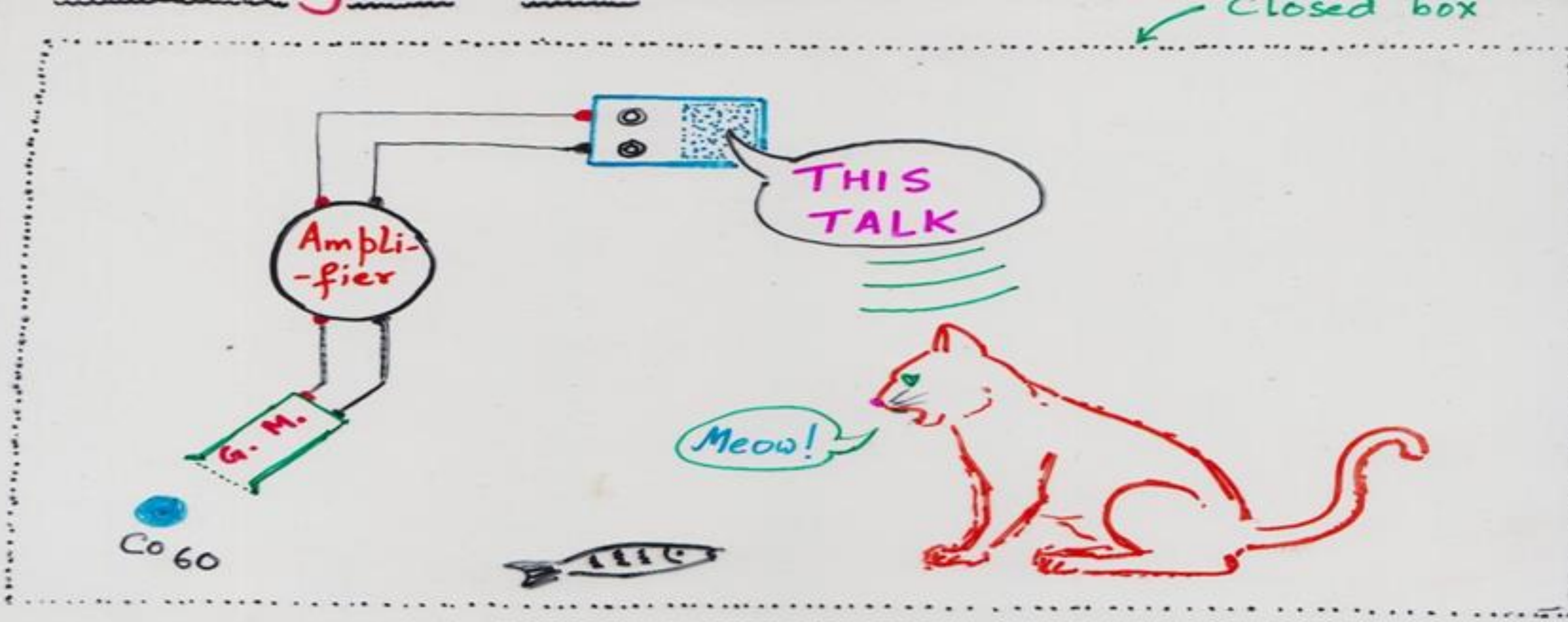
A. N. MITRA MEMORIAL MEETING AND LECTURE, April 14-15, 2025.

Image Credit:

https://www.bibhudevmisra.com/2019/02/the-yali-symbol-on-indus-seal-and-its.htm

SHARDULA : Superposition of distinct animals



G.M. counter click $\longrightarrow$ "This talk" switched on $\rightarrow$ Cat dies

No click $\longrightarrow$ "No talk" $\longrightarrow$ Cat is alive

State of the Cobalt-60 : $\frac{1}{\sqrt{2}}\left\{ |\text{decay}\rangle + |\text{no decay}\rangle \right\}$

After the Co_{60} is put in the box :

$$|\text{State}\rangle = \frac{1}{\sqrt{2}}\Big\{ |\text{decay}\rangle |\text{click}\rangle |\text{talk}\rangle |\text{cat dead}\rangle + |\text{no decay}\rangle |\text{no click}\rangle |\text{no talk}\rangle |\text{cat alive}\rangle \Big\}$$

Paradox : When does the $|\text{state}\rangle$ collapse to either to $|\text{decay}\rangle \ldots\ldots |\text{cat dead}\rangle$ or to $|\text{no decay}\rangle \ldots\ldots |\text{cat alive}\rangle$?

IN THE ABSENCE OF ANY MEASUREMENT, THE STATE EVOLVES ACCORDING TO THE SCHRODINGER'S EQUATION, IN A DETERMINISTIC WAY

$$i\hbar \frac{d}{dt} |\psi\rangle = \hat{H} |\psi\rangle$$

Fourth Postulate (case of a discrete non-degenerate spectrum): When the physical quantity $\mathcal{A}$ is measured on a system in the *normalized* state $|\psi\rangle$, the probability $\mathcal{P}(a_n)$ of obtaining the *non-degenerate* eigenvalue a_n of the corresponding observable A is:

$$\mathcal{P}(a_n) = |\langle u_n | \psi \rangle|^2$$

where $|u_n\rangle$ is the normalized eigenvector of A associated with the eigenvalue a_n.

*** A MEASUREMENT TAKES PLACE WHEN AN APPARATUS INTERACTS WITH THE SYSTEM UNDER OBSERVATION**

*** THIS INTERACTION IS REPRESENTED BY AN INTERACTION HAMILTONIAN**

*** APPARATUS + SYSTEM SHOULD EVOLVE ACCORDING TO SCHRODINGER EQUATION**

THE INTERACTION BETWEEN THE SYSTEM UNDER OBSERVATION AND THE APPARATUS LEADS TO:

*** AN ENTANGLEMENT BETWEEN THE STATES OF THE SYSTEM AND THE APPARATUS**

*** EXCHANGE OF ENERGY BETWEEN THE SYSTEM'S FEW DEGREES OF FREEDOM AND INNUMERABLE DEGREES OF FREEDOM OF THE APPARATUS →**

(a) Energy-time uncertainty

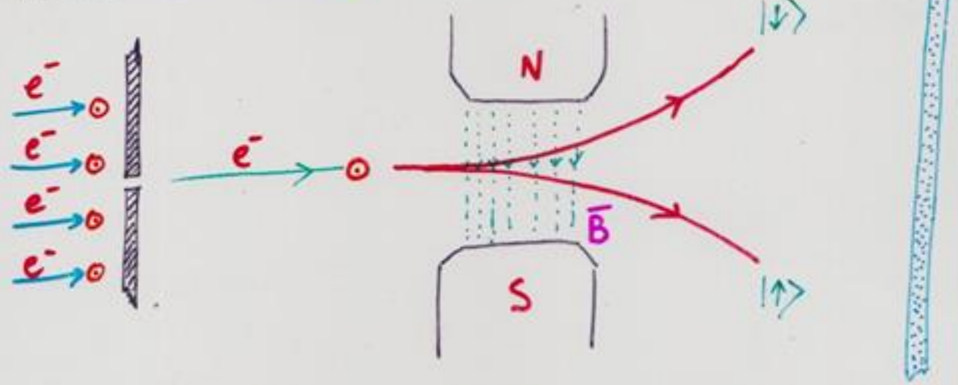
(b) Energy fluctuation → General relativity → Metric fluctuation → Fluctuation in the proper time

(c) Increase in Entropy

*** GRAVITY AND QUANTUM THEORY: Is there a Planck number of degrees of freedom?**

Planck mass/electron mass ~ 10^(22)

Measurement theory revisited



$$\psi(\vec{r})\cdot|\odot\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\rangle + |\downarrow\rangle\Big]\psi(\vec{r}) \longrightarrow |\uparrow\rangle\psi_-(\vec{r}) + |\downarrow\rangle\psi_+(\vec{r})$$

When does the **measurement** take place?

- Inhomogenous magnetic field **alone** does **not** complete the measurement process
- Detection at the (photographic plate, proportional counter, ...) very **crucial**

$$\therefore\ \psi(\vec{r})\cdot|\odot\rangle\cdot|D\rangle \longrightarrow \psi_-(\vec{r})\,|\uparrow\rangle\,|\text{atoms}^*_-\rangle + \psi_+(\vec{r})\,|\downarrow\rangle\,|\text{atoms}^*_+\rangle$$

Note: $|\text{atoms}^*_\pm\rangle = e^{i\chi_\pm}\,|\text{atoms}_\pm\rangle$

where $\chi_\pm$ are **random phases** $\in [0, 2\pi)$.

- Interaction between $\bar{e}$ and $\bar{B}$ is deterministic
- Interaction between e^- and the detector **atoms** is (1) over a **compact spatial volume**, (2) on short time scales, (3) with a large number of atoms and (4) **causing** transfer of energy from e^- to a large **number** of degrees of freedom (atoms, photons, gravitons, ...)

Atomic Size $\sim 10^{-8}$ cm , $\Delta t \sim \frac{10^{-8}\,cm}{3\times10^{10}\,cm\,s^{-1}} \sim 3\times10^{-19}\,s$

$\Delta E \sim \frac{h}{\Delta t} \sim 2\times10^{-8}$ erg ; $\delta g_{00} \sim \frac{2\delta\phi}{c^2} \sim \frac{2G\,\delta E}{c^4 l} \sim 3\times10^{-49}$

Proper time intervals, $\Delta\tau \sim \Delta\tau_0 (1 \pm 10^{-49})$

Violation of energy conservation, $\delta E \sim mc^2 \times 10^{-49}$

Uncertainty in phases $\delta\phi \sim \delta\left(\frac{E\times\tau}{\hbar}\right)$

$$\Psi(\bar{r}).|0\rangle.|D\rangle \longrightarrow |\Psi_-\rangle\,|\uparrow\rangle.\,|atoms^*_-(g_{\mu\nu-}, g_{\mu\nu+})\rangle$$
$$+\,|\Psi_+\rangle|\downarrow\rangle\,|atoms^*_+(g_{\mu\nu-}, g_{\mu\nu+})\rangle$$

Quantum system : $|x\rangle_q$; $\hat{x}_q|x\rangle_q = x|x\rangle_q$

Apparatus : Pointer states, $\hat{X}_a|x\rangle_a = x|x\rangle_a$

$\hat{P}_a$ = Canonical conjugate to $\hat{X}_a \Rightarrow [\hat{X}_a, \hat{P}_a] = i\hbar$

Measurement process: $\hat{H}_{int} = g\,\hat{x}_q\,\hat{P}_a$

Suppose : At $t=0$: $|\Psi(0)\rangle = |x\rangle_q \otimes |0\rangle_a$

Measurement $\rightarrow |\Psi(t)\rangle = e^{-i\hat{H}_{int}t}|\Psi(0)\rangle$

$$\therefore |\Psi(t)\rangle = e^{-igt\,\hat{x}_q\hat{P}_a}|x\rangle_q \otimes |0\rangle_a$$

$$= e^{-igt\,x\,\hat{P}_a}|x\rangle_q \otimes |0\rangle_a$$

$$= |x\rangle_q \otimes |gt\,x\rangle_a$$

Reading off the pointer position $\rightarrow$ Position of the quantum system

But if, at $t=0$: $|\Psi(0)\rangle = (c_1|x_1\rangle_q + c_2|x_2\rangle_q)\otimes|0\rangle_a$

Measurement $\Rightarrow |\Psi(t)\rangle = e^{-igt\,\hat{x}_q\hat{P}_a}(c_1|x_1\rangle_q + c_2|x_2\rangle_q)\otimes|0\rangle_a$

Linear operator $\Rightarrow \Psi(t) = c_1 e^{-igt\,\hat{x}_q\hat{P}_a}|x_1\rangle_q \otimes |0\rangle_a +$

$$+\, c_2 e^{-igt\,\hat{x}_q\hat{P}_a}|x_2\rangle_q \otimes |0\rangle_a$$

$= c_1|x_1\rangle_q \otimes |gt\,x_1\rangle_a + c_2|x_2\rangle_q|gt\,x_2\rangle_a \rightarrow$ Entangled!

Couple the apparatus ...

$$\left[c_1|x_1\rangle_q \otimes |gtx_1\rangle_a + c_2|x_2\rangle_q \otimes |gtx_2\rangle_a\right] \otimes |\theta\rangle$$

$$\longrightarrow c_1|x_1\rangle_q \otimes |gtx_1\rangle_a \otimes |\theta_{x_1}\rangle + c_2|x_2\rangle_q \otimes |gtx_2\rangle_a \otimes |\theta_{x_2}\rangle$$

and so on .....

Source of the problem

- Principle of linear superposition: $c_1|\psi_1\rangle + c_2|\psi_2\rangle$
- Observables $\Longleftrightarrow$ Linear operators (self-adjoint)

  Time evolution: $i\dfrac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

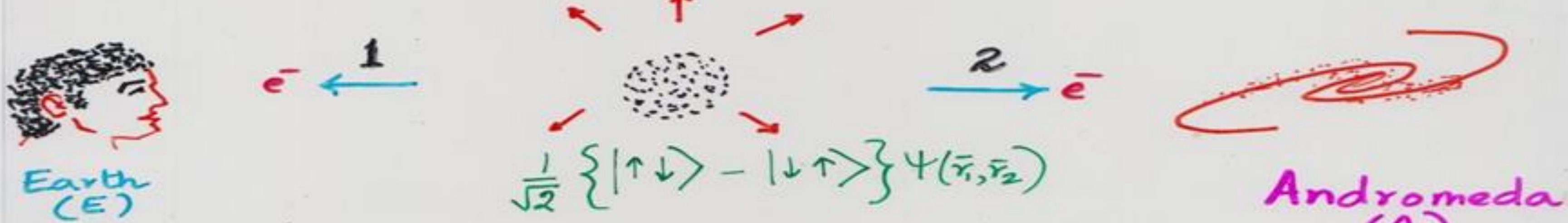
  Equivalently, $|\psi(t)\rangle = e^{i\hat{H}t}|\psi(0)\rangle$

$$\therefore c_1|\psi_1(0)\rangle + c_2|\psi_2(0)\rangle \longrightarrow e^{i\hat{H}t}\left(c_1|\psi_1(0)\rangle + c_2|\psi_2(0)\rangle\right) = c_1|\psi_1(t)\rangle + c_2|\psi_2(t)\rangle$$

- Where from Born 'interpretation?

  $|\psi_i\rangle$ with probability $|c_i|^2$

  After all, measurement is an interaction!



$$\frac{1}{\sqrt{2}}\left\{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right\}\Psi(\bar{r}_1,\bar{r}_2)$$

After the decay: State $\rightarrow \frac{1}{\sqrt{2}}\left\{ \overset{1}{|\uparrow,\psi_E\rangle}\,\overset{2}{|\downarrow,\phi_A\rangle} - \underset{1}{|\downarrow,\psi_E\rangle}\,\underset{2}{|\uparrow,\phi_A\rangle} \right\}$

Measurement of spin of electron 1 on Earth:

Collapse of the state vector to:

Either $|\uparrow,\psi_E\rangle|\downarrow,\phi_A\rangle$ or $|\downarrow,\psi_E\rangle|\uparrow,\phi_A\rangle$

Changing the state at Andromeda through Local measurements on Earth !!

Spooky action-at-distance ?

States of 1 are correlated with states of 2 $\longleftarrow$ Unique feature arising from the Hilbert space structure in Quantum Mech.

Ways out • Copenhagen interpretation (...)

• Many-worlds interpretation (Everett, Wheeler, DeWitt, Hartle, ...)

• Decoherence (Zurek, ...)

• Gravity induced collapse (Penrose, Diosi, Christian..)

Time scale for collapse $\sim \dfrac{\hbar}{\Delta E_G}$

$$\Delta E_G \sim 4\pi G \int d^3r_1 \int d^3r_2 \, \frac{[\rho_1(\bar{r}_1) - \rho_2(\bar{r}_1)][\rho_1(\bar{r}_2) - \rho_2(\bar{r}_2)]}{|\bar{r}_1 - \bar{r}_2|}$$

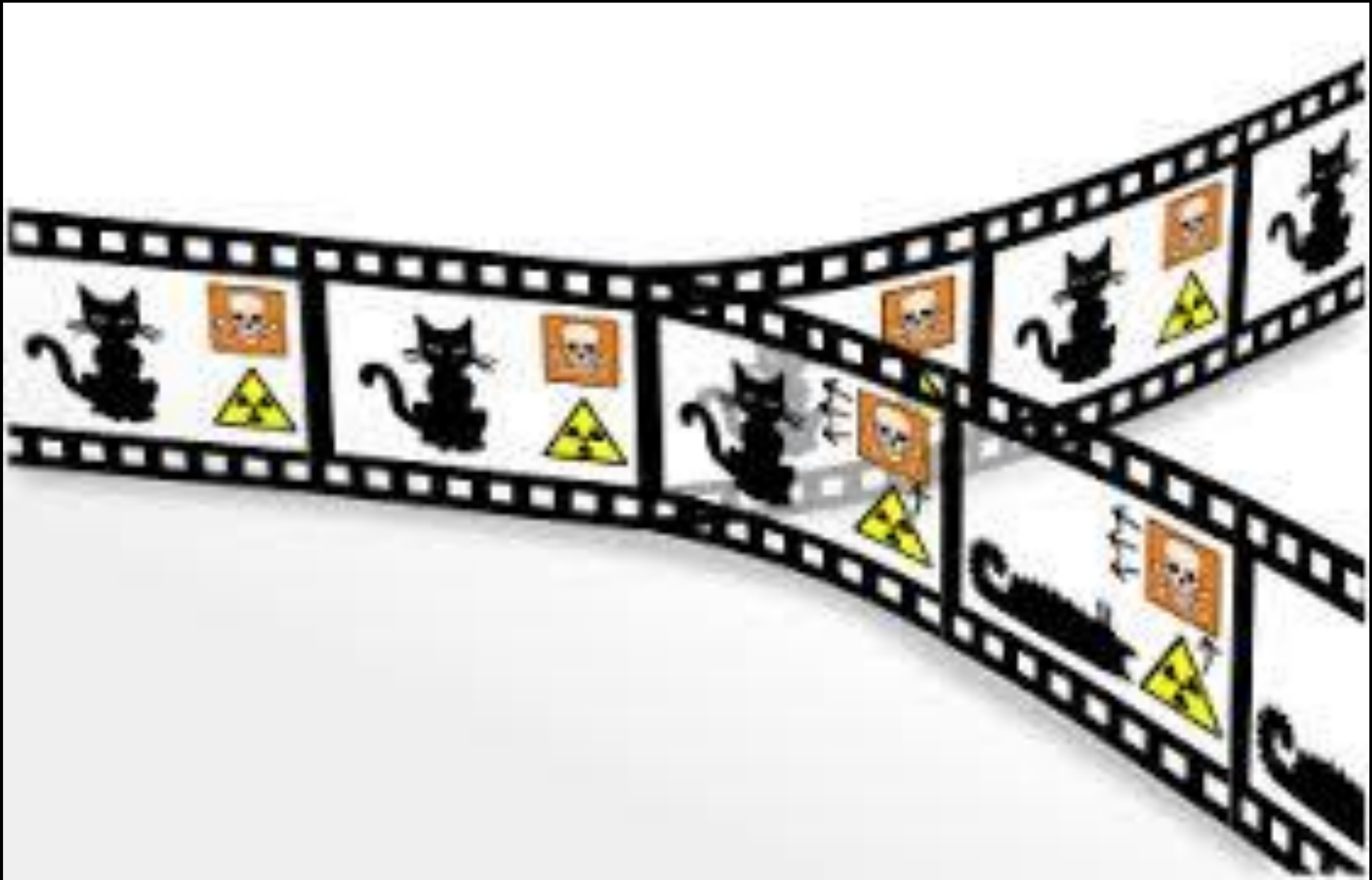
$|\psi_1\rangle \longrightarrow \rho_1 \qquad |\psi_2\rangle \longrightarrow \rho_2$

For $\quad c_1|\psi_1\rangle + c_2|\psi_2\rangle \xrightarrow{\text{Collapse}} \begin{cases} \longrightarrow |\psi_1\rangle \\ \longrightarrow |\psi_2\rangle \end{cases}$

$\Delta E_G \sim$ Gravitational self-energy of the difference in the mass distribution

Problems : • What is the mechanism?

• Why is the collapse probility $|c_i|^2$



Many Worlds Theory : **But, what about Born rule?**

Role of Gravity

1. $ds^2 = g_{\mu\nu} dx^\mu dx^\nu \Rightarrow$ Proper time: $\sqrt{g_{00}}\, dt$

2. Globally conserved energy and momentum require Killing vectors.

Suppose, the metric is stationary.

$$g_{\mu\nu}(x^0 + \delta x^0, x^i) = g_{\mu\nu}(x^0, x^i) \Rightarrow \xi^\mu = (1,0,0,0)$$

$$\frac{d}{ds}\left(g_{\mu\nu} p^\mu \xi^\nu\right) = 0 \Rightarrow \frac{E}{c} \equiv g_{\mu 0} p^\mu \text{ is conserved.}$$

What happens when the metric is perturbed?

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \delta g_{\mu\nu} \Rightarrow \frac{d}{d\bar{s}}\left(p^\mu \xi_\mu\right) = \left(p^\mu \xi_\mu\right)_{;\bar{\nu}} \frac{dx^{\nu}}{d\bar{s}}$$

$$\bar{\Gamma}^\lambda_{\mu\nu} \approx \Gamma^\lambda_{\mu\nu} + \frac{1}{2} g^{\lambda\sigma}\left[\delta g_{\sigma\mu,\nu} + \delta g_{\sigma\nu,\mu} - \delta g_{\mu\nu,\sigma}\right]$$

$$\frac{d}{d\bar{s}}\left(p^\mu \xi_\mu\right) \approx \frac{-1}{2} mc\, \xi^\sigma \left\{ \left[\delta g_{\sigma\mu,\nu} + \delta g_{\sigma\nu,\mu} - \delta g_{\mu\nu,\sigma}\right] - 2\,\delta g_{\sigma\lambda}\Gamma^\lambda_{\mu\nu} \right\} \times \frac{dx^\mu}{d\bar{s}}\frac{dx^\nu}{d\bar{s}}$$

For non-relativistic particles, in weak gravity:

$$\frac{d}{dt}\left(p^\mu \xi_\mu\right) \approx \frac{-1}{2} mc^2\, \xi^\sigma \left\{ \left[2\,\delta g_{0\sigma,0} - \delta g_{00,\sigma}\right] - 2\,\delta g_{\sigma\lambda}\Gamma^\lambda_{00} \right\}$$

With $\xi^\mu = (1,0,0,0)$: $\frac{d}{dt}\left(E/c\right) \approx \frac{-1}{2} mc^2 \left[\delta g_{00,0} - 2\delta g_{0\lambda}\Gamma^\lambda_{00}\right]$

$$\therefore \quad \frac{dE}{dt} \approx -m\frac{d}{dt}(\delta\phi) \qquad \text{since } \delta g_{00} = \frac{2\,\delta\phi}{c^2}$$

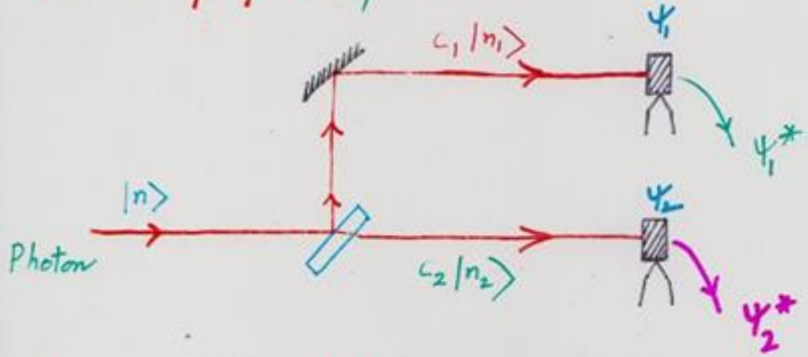
→ Violation of Energy conservation

Proper time: $d\bar{\tau} = \sqrt{\bar{g}_{00}}\, dt = \sqrt{g_{00} + \delta g_{00}}\; dt$

$$\Rightarrow \; d\bar{\tau} \approx d\tau \left(1 + \frac{\delta\phi}{2\phi}\right)$$

But, in QM : Time evolution depends on E and time

A simplified problem:



Initial state: $|n\rangle \cdot \psi_1 \cdot \psi_2 = (c_1|n_1\rangle + c_2|n_2\rangle) \cdot \psi_1 \cdot \psi_2$

Final state: $c_1|n_1'\rangle \psi_1^*\left(t(\psi_1^*, \psi_2^*), \psi_2^*\right) \cdot \psi_2'(\psi_1^*)$

$+ \; c_2|n_2'\rangle \psi_1'(\psi_2^*) \cdot \psi_2^*\left(t(\psi_1^*, \psi_2^*), \psi_1^*\right)$

There is cross talking!

- Linear evolution cannot be right
- Uncertainty principle ⇒ Random relative phase
- General relativity necessitates non-linear evolution

Non-Linear Schrödinger eq. (Grigorenko, 1995):

$$i\frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle + \left[1 - |\Psi(t)\rangle\langle\Psi(t)|\right]\hat{U}|\Psi(t)\rangle$$

$$\hat{U} \equiv -ig\sum_{n=1}^{2}\frac{|\langle\Psi(0)|\phi_n\rangle|^2}{\ln|x_n/2\pi|}\,|\Phi_n\rangle\langle\Phi_n|$$

$$|\Psi(0)\rangle = c_1(0)|n_1\rangle . |\psi_1\rangle . |\psi_2\rangle + c_2(0)|n_2\rangle |\psi_1\rangle |\psi_2\rangle$$

$$|\Psi(t)\rangle = c_1(t) \times e^{i\chi_1}|\Phi_1\rangle + c_2(t) \times e^{i\chi_2}|\Phi_2\rangle$$

$$\frac{dc_1(t)}{dt} = \left[-iE_1 + g\sum_{n=1}^{2} \frac{|c_n|^2 |c_n(t)|^2}{\ln|\chi_n/2\pi|} - g\frac{|c_1|^2}{\ln|\frac{\chi_1}{2\pi}|}\right] c_1(t)$$

$$\frac{dc_2(t)}{dt} = \left[-iE_2 + g\sum_{n=1}^{2} \frac{|c_n|^2 |c_n(t)|^2}{\ln|\chi_n/2\pi|} - g\frac{|c_2|^2}{\ln|\frac{\chi_2}{2\pi}|}\right] c_2(t)$$

where $\hat{H} \quad |\Phi_i\rangle = E_i \quad |\Phi_i\rangle, \ i = 1, 2.$

Solutions : $$c_i(t) = \frac{e^{-iE_i t} \ e^{-\frac{g|c_i|^2 t}{\ln|\chi_i/2\pi|}}}{\left(e^{-\frac{2g|c_1|^2 t}{\ln|\chi_1/2\pi|}} . |c_1|^2 + e^{-\frac{2g|c_2|^2 t}{\ln|\chi_2/2\pi|}} . |c_2|^2\right)^{1/2}}$$

for $i=1,2$

$\ln|\chi_i/2\pi|$ are negative valued, random numbers.

$$\therefore |c_i(t)|^2 = \frac{e^{-2g\frac{|c_i|^2 t}{\ln|\chi_i/2\pi|}}}{e^{-\frac{2g|c_1|^2 t}{\ln|\chi_1/2\pi|}} |c_1|^2 + e^{-\frac{2g|c_2|^2 t}{\ln|\chi_2/2\pi|}} |c_2|^2}$$

the entire system photon + detector 1 + detector 2.

2. $|\Phi_1\rangle = |n_1'\rangle \psi_1^* \cdot \psi_2'$

$|\Phi_2\rangle = |n_2'\rangle \psi_1' \cdot \psi_2^*$

3. $\therefore |\Psi(0)\rangle = c_1(0)|n_1\rangle \psi_1 \cdot \psi_2 + c_2(0)|n_2\rangle \psi_1 \psi_2$

and $|\Psi(t)\rangle = c_1(t)|\Phi_1\rangle e^{i\chi_1} + c_2(t)|\Phi_2\rangle e^{i\chi_2}$

Characterstics of the solution:

1. $|c_1(t)|^2 + |c_2(t)|^2 = 1$ $\leftarrow$ Probability interpretation

2. If $\dfrac{-|c_i(0)|^2}{\ln|\frac{\chi_i}{2\pi}|} < \dfrac{-|c_j(0)|^2}{\ln|\chi_j/2\pi|}$ then $|c_j(t)|$ grows

at the expense of $|c_i(t)|$

3. $$|c_j(t)|^2 = \frac{1}{1 + e^{-2gt\left(\frac{|c_i|^2}{(\ln|\frac{\chi_i}{2\pi}|)} - \frac{|c_j|^2}{\ln|\chi_j/2\pi|}\right)}} \xrightarrow{t\to\infty} 1$$

4. Phases χ_i's are random because of a combination of fluctuating metric, large degrees and $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$.

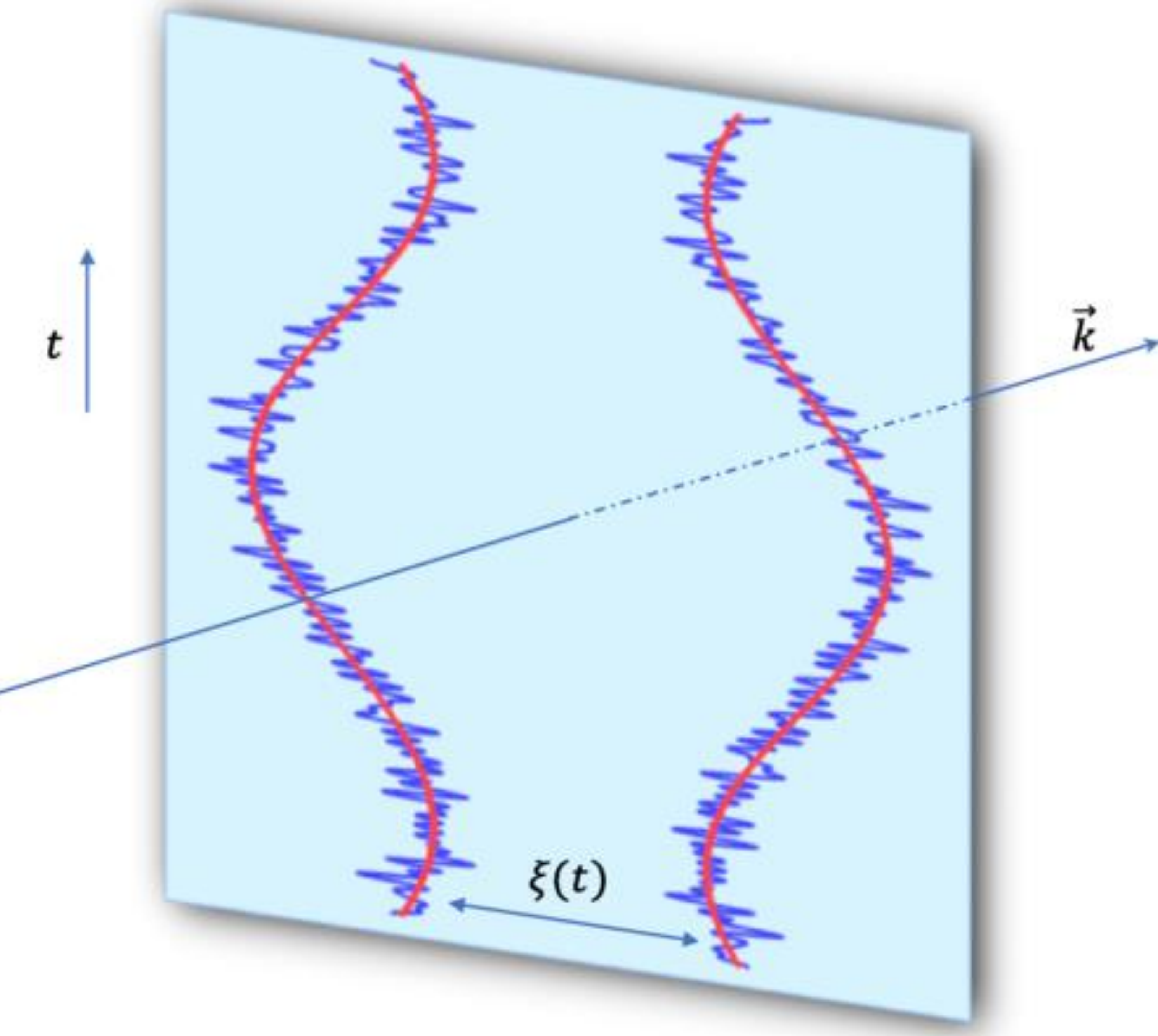
Quantum Gravity effects and gravitational wave (GW) detectors

(Amelino-Camelia (1999);…; Dyson (2013); …; Parikh, Wilczek and Zahariade, Phys. Rev.Lett. (2021);….)

* Quantum GW effects on the geodesic deviation equation

* Characteristic Quantum GW noise and LIGO

The noise depends on the specific quantum state of the GW



Gravitational Wave Quantum effects on the separation between two geodesics

Image Credit:

Parikh, Wilczek and Zahariade, Phys. Rev. Lett., 2021)


Thank You!