UNIVERSAL $1/r^2$ POTENTIALS

at short and long range

avoids collapse holds quasi-bound states

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Ph. D. U of Chicago, 1970, Ugo Fano, 3-body Coulomb threshold laws, Doubly-excited states of H^{--} and He, ….. nuclear 3-body ^{20}C: $^{18}C + n + n$, ….
B.Sc. (Hons) 1964, M. Sc. 1966, U of Delhi
A. N. Mitra, Mathematical Physics, Quantum Field Theory

F. C. Auluck, S. N. Biswas, L. S. Kothari, V. S. Mathur, M. L. Mehta, N. Panchapakesan,
R. K. Pathria, R. P. Saxena, P. K. Srivastava, …

TIFR, Nov 2010, N-N Interactions and Nuclear Many-Body Problem

Vary Mitra Warke

Efimov, Faddeev, Belyaev

Themes:

Quantum binding, $1/r^2$ potentials at small and large r

Attractive dipole $1/r^2$ potential - bound states

Resonances: asymmetric “Fano” profiles

Scattering length, and its tuning

Neutron rich species: $n + n + A$

Universal aspects: atomic, molecular, nuclear

Angular Kinetic Energy $\ell^2/2mr^2$

Repulsive barrier: at small r avoids collapse, at large r holds low energy particles from penetration and/or escape

Classical Physics: Collapse for straight line $\ell = 0$

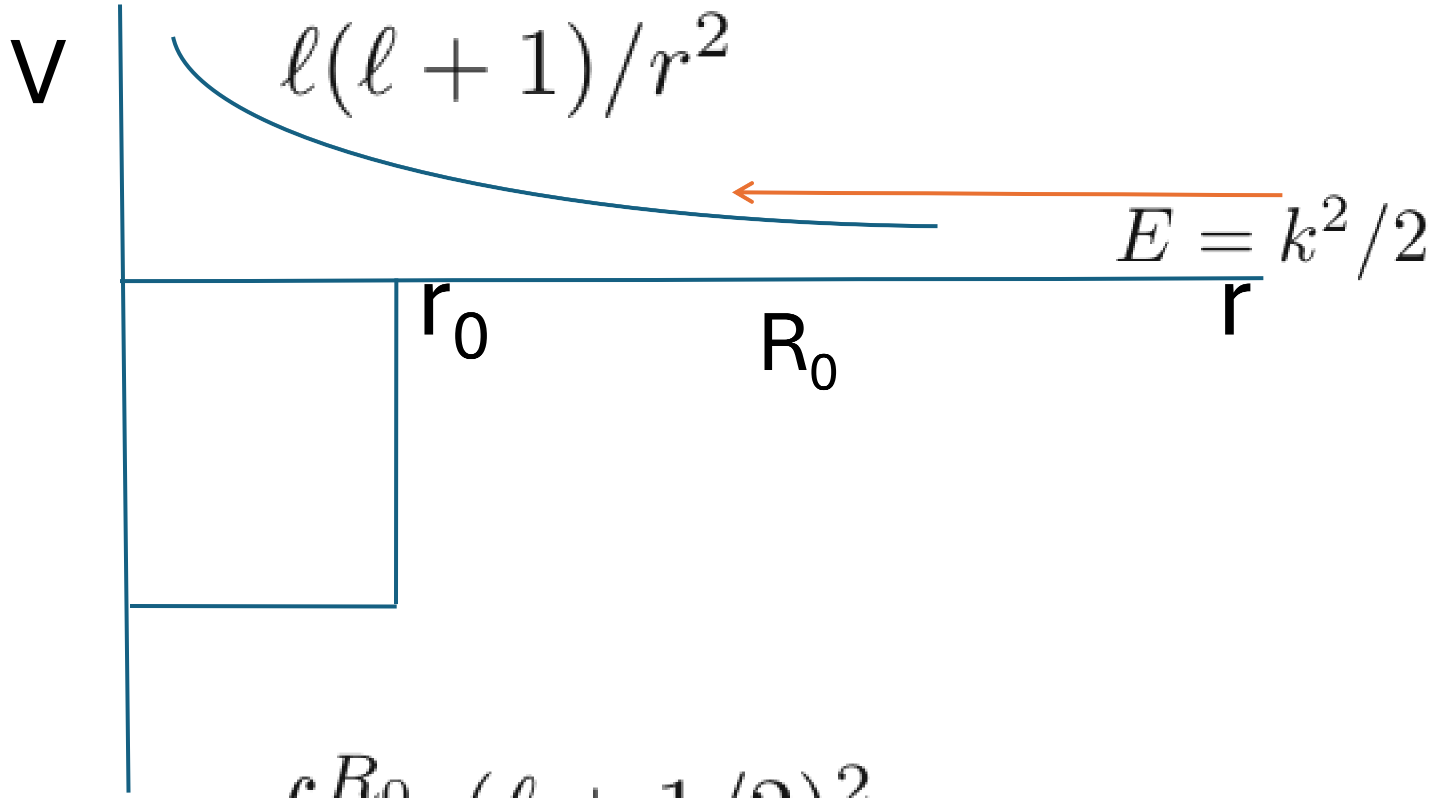
Quantum Physics, $\hbar$ always an $\hbar^2/mr^2$

radial kinetic energy

Stability of H atom $\hbar^2/(2mr^2) - e^2/r$ Bohr radius a_0

Wigner Threshold Law $k^{2\ell+1}$ tunneling at large r

Tunneling through angular potential barrier



$$\exp(-2\int_{r_0}^{R_0}[\frac{(\ell+1/2)^2}{r^2} - k^2]^{1/2} dr)$$

$k^{2\ell+1}$ elastic scattering phaseshift

inelastic cross-section

Attractive $-a/2r^2$ potential, effective complex angular momentum

$$\lambda = -\frac{1}{2} + i\alpha \qquad \alpha = \sqrt{a - \frac{1}{4}}$$

Number of bound states: Infinite number for long range

$$\hbar^2/2mr^2 \quad \text{vs} \quad -e^2/r$$

$1/r^2$ the dividing falloff between long/short range, infinite/finite #

for $1/r^2$ itself, the strength parameter a decides

Infinite sequence described by sin ( ) = 0 => eigenvalues

$\epsilon = -\kappa^2/2$

Coulomb : $\pi Z/\kappa = n\pi, \epsilon_n = -Z^2/2n^2$

Dipole bound states: molecules (OH, H_2O), H atom in n > 1 :

$$\alpha \ln(2/\kappa) - arg\Gamma(1 - i\alpha) = (n + 1/2)\pi \qquad \epsilon_n = \epsilon_0 e^{-2n\pi/\alpha}$$

D-dimensional Laplacian:

$$\frac{\hbar^2}{2M}\ \{ -\frac{d^2}{dR^2} + \frac{D-1}{R}\frac{d}{dR} + \frac{\Lambda^2(angles)}{R^2}\ \}$$

radial angular

$$\Lambda^2 : \lambda(\lambda + D - 2),\ D = 2 : m^2,\ D = 3 : \ell(\ell+1)$$

N – body problem : D = 3(N-1) three-body: 6

$R^2 = r_1^{\,2} + r_2^{\,2} + \ldots..$ R overall size of the system, remainder angles

$$R = (\sum_i m_i r_i^2/M)^{1/2} \qquad \Psi \to R^{(D-1)/2}\Psi$$

$$-\frac{d^2}{dR^2} + \frac{(D-1)(D-3)}{4R^2}$$

radial kinematic potential

D = 2 attractive

D > 3 repulsive

Universal, reflecting all but one R coordinate, “hard-sphere” at small R, long range tail

Resonances in physics: symmetric Lorentzian shapes

Breit-Wigner in nuclear and particle physics

$$\sigma = \frac{A}{(E-E_r)^2 + (\Gamma/2)^2} = \frac{\sigma_0}{1+\epsilon^2}$$

$$\epsilon \equiv (E-E_r)/(\Gamma/2)$$

Fano (1961) resonance as interference between two pathways

$$\sigma = \sigma_0 \frac{(q+\epsilon)^2}{(1+\epsilon^2)}$$

Fano profile, asymmetric, zero at $\epsilon = -q$

Reduces to Lorentzian for $q \longrightarrow \infty$

Elastic scattering $\sigma = (4\pi/k^2)\sin^2\delta$

δ_a background

$$\sin\delta = \sin(\delta_a + \delta - \delta_a) = \sin\delta_a\cos(\delta - \delta_a) + \cos\delta_a\sin(\delta - \delta_a)$$

$$\sin^2\delta = \sin^2\delta_a \frac{[\cot(\delta - \delta_a) + \cot\delta_a]^2}{1 + \cot^2(\delta - \delta_a)}$$

$$q = -\cot\delta_a, \;\; \epsilon = -\cot(\delta - \delta_a)$$

$\delta_a = 0$, $\quad q \to \infty \quad$ reduces to Lorentzian

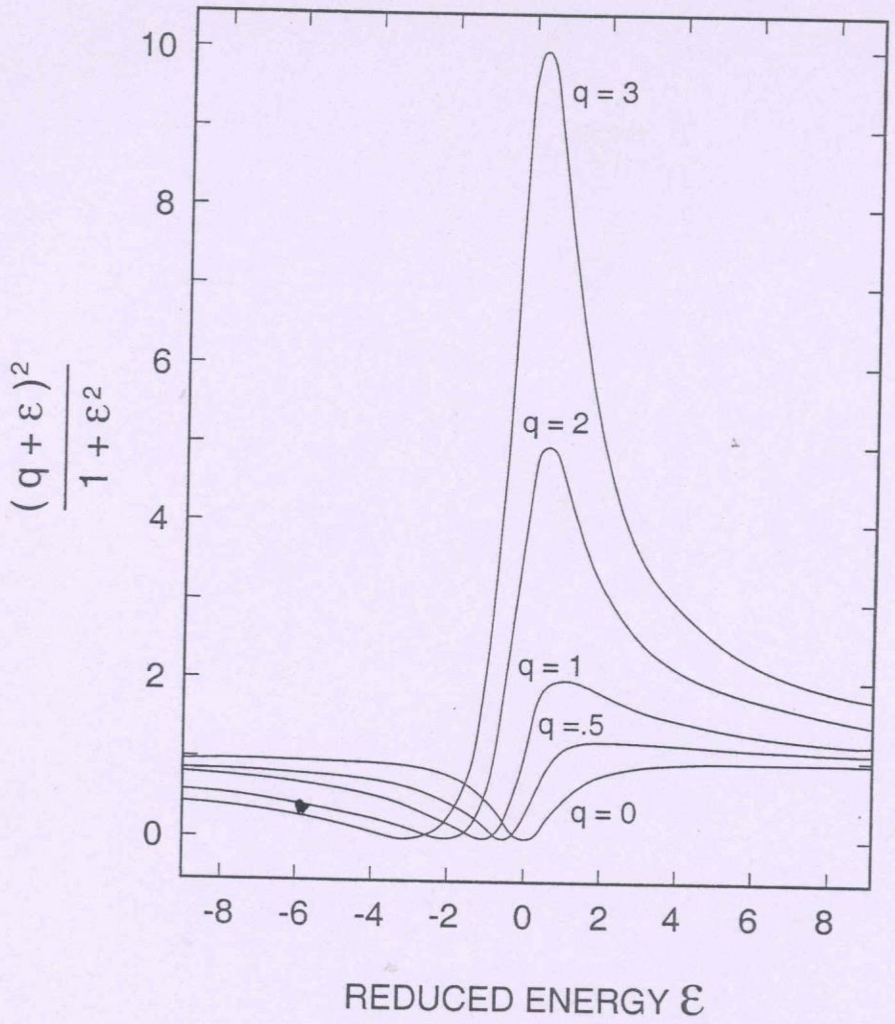
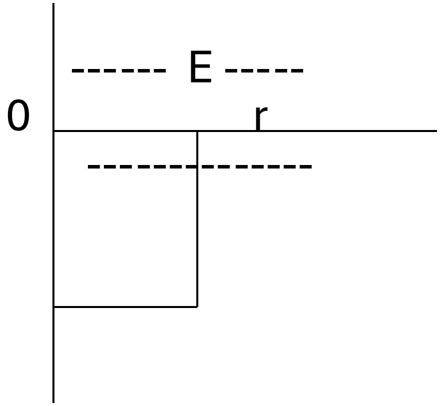


Figure 5.13. Beutler-Fano profiles with different values of the Fano q-parameter. From U. Fano, Phys. Rev. **124**, 1866 (1961).

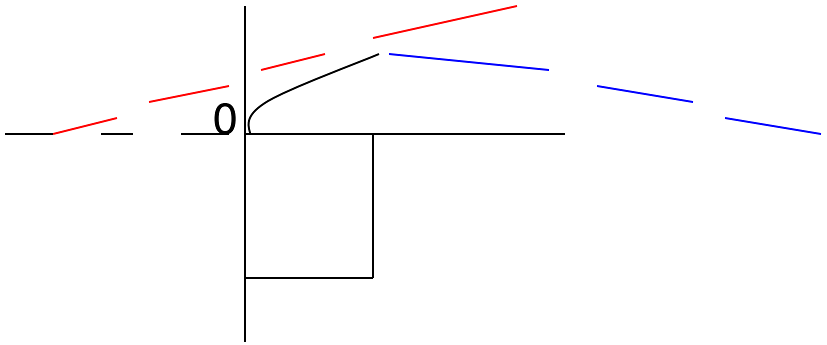
Scattering length: s-wave

$$[-\frac{d^2}{dr^2}+\frac{\ell(\ell+1)}{r^2}+2V(r)-2E]u(r)=0$$



$$\ell=0, V=0, E\approx 0$$

$$u=N(1-r/a)$$



$a<0$ only scattering state

$a>0$ possible bound state as well

$|a|\to\infty$ boundary for binding

weak binding: very large spread of wave function, e.g. deuteron, H^{--}

V. Efimov, Phys. Lett. **33**B, 563 (1970) L.H.Thomas (1935)

Two-body with range r_0
Effective $1/R^2$ potential
as scattering length
gets large

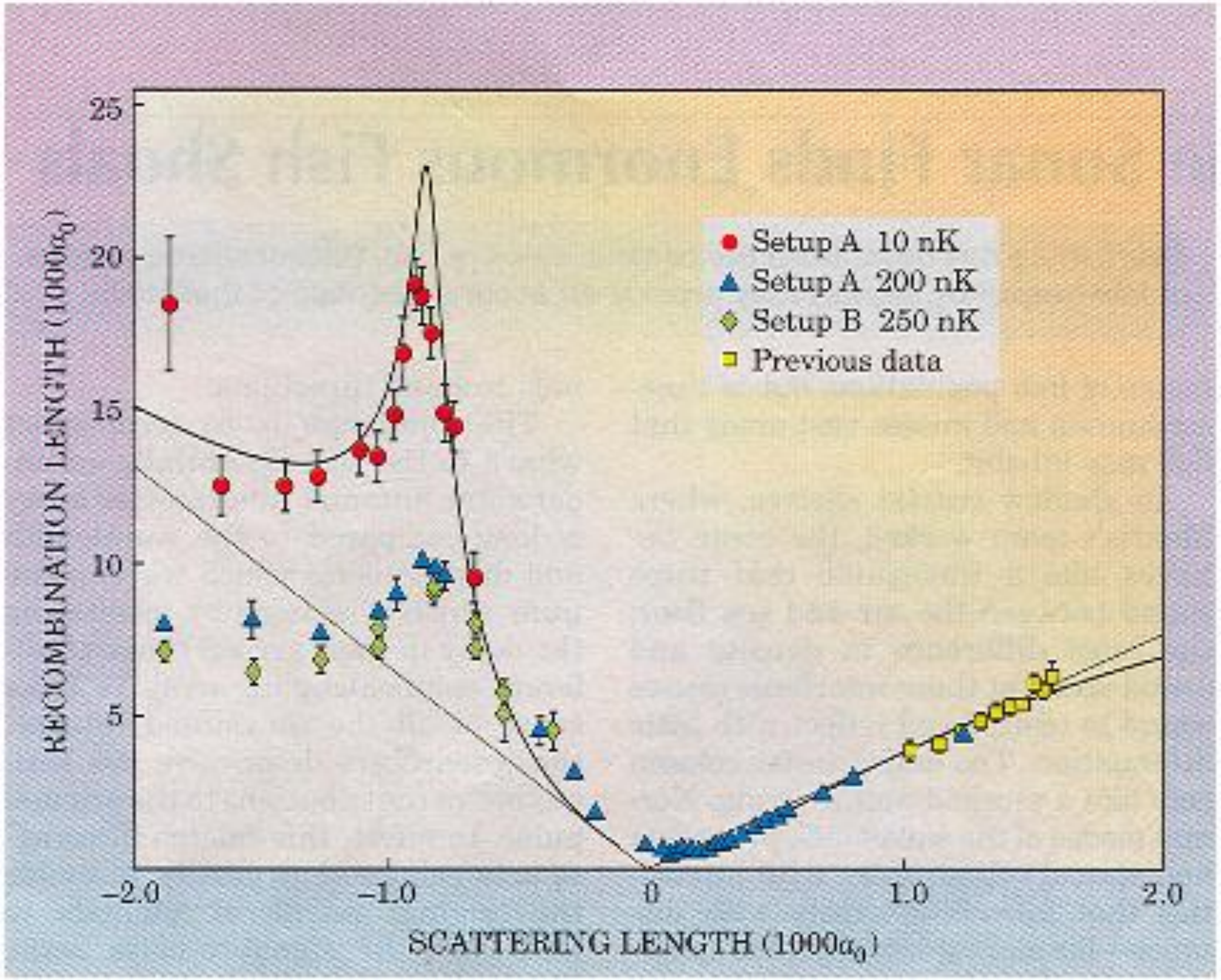
$$N = \frac{1}{\pi} \ln \frac{|a|}{r_0}$$

As two-body binding increases, Efimov states disappear

Excited state of helium trimer $^4He_3$; $^4He_2$ weakly bound
Cold atom traps, magnetic tuning of atom-atom scattering length. Cesium atoms through van der Waals, $r_0 = 100\ a_0$ Tune a between -2500 and 1600 a_0 10 nK

Kraemer et al, Nature **440**, 315 (2006); Esry-Greene, p.289

Efimov states of Cesium trimers



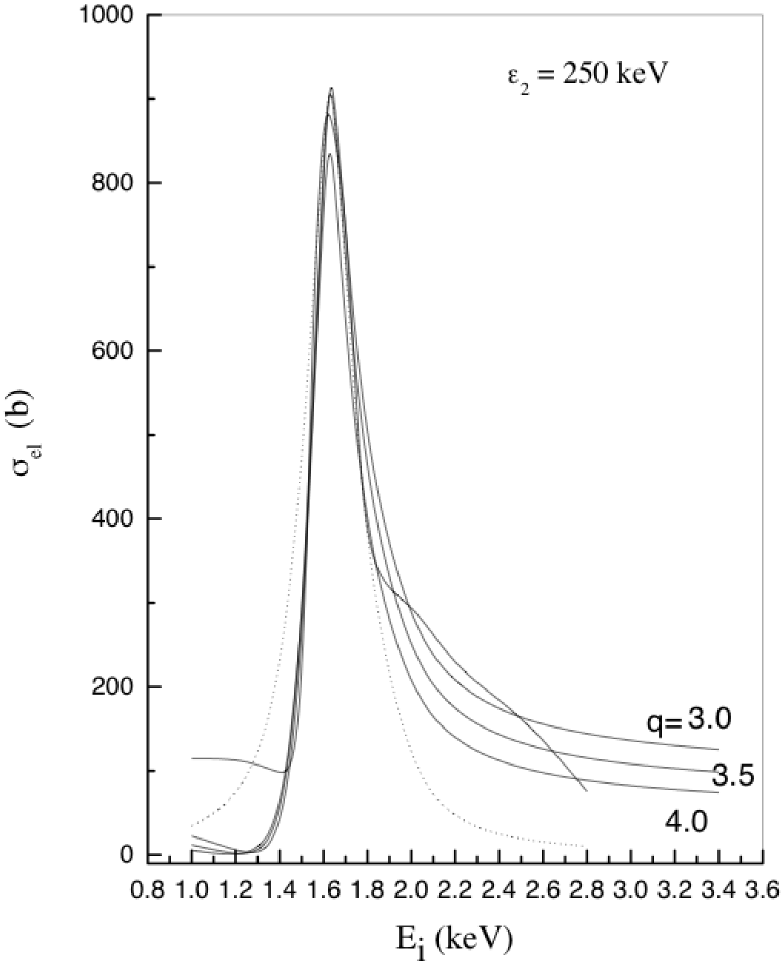
Kraemer et al, Nature **440**, 315 (2006), Esry-Greene, p.289
C. H. Greene, Phys. Today **63** (3), 40 (2010)

I.Mazumdar, ARPR, V.S.Bhasin, Phys.Rev.Lett. **97**,062503 (2006)

neutron-rich, halo nuclei. $^{20}C : n + n + {}^{18}C$

two-body energy ^{19}C tuned

elastic $n + {}^{19}C$ cross-section for energy E_i



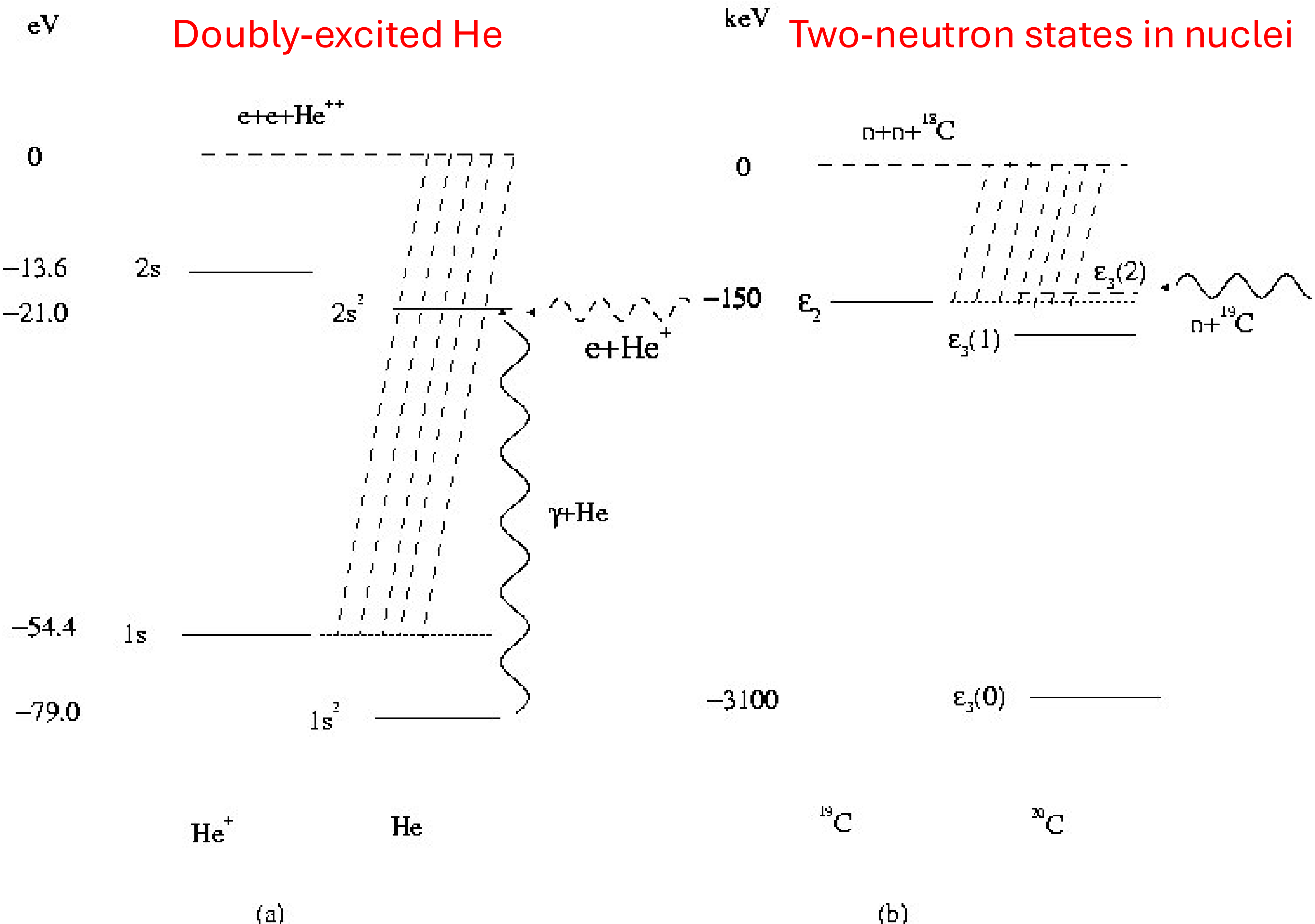
E_r =1.63 keV

Γ = 0.25 keV

q = 4

Doubly-excited He

Two-neutron states in nuclei

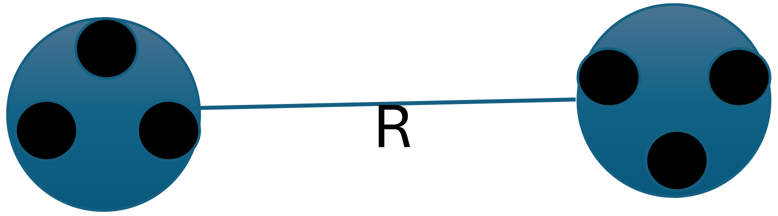


N – N Potential   Repulsive Barrier/Core at small R

1969 U. Fano Wannier 3-body threshold law with Coulomb

$\longrightarrow$ $H^+ + e + e$

Repulsive $1/R^2$ Potential
at small R



Not really a two-body but an effective potential of a many-body system

Number of particles not defined, a conjugate phase as in BCS

Born approximation: $\sigma_{el} \sim [E^2 \ln \vec{p}^2 - \vec{p}^2]/\vec{p}^2$

$$E^2 = \vec{p}^2 + m^2$$