Ringdown of Rotating Black Holes

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Outline

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1. Introduction
2. EdGB BHs
3. Ringdown
4. Conclusions

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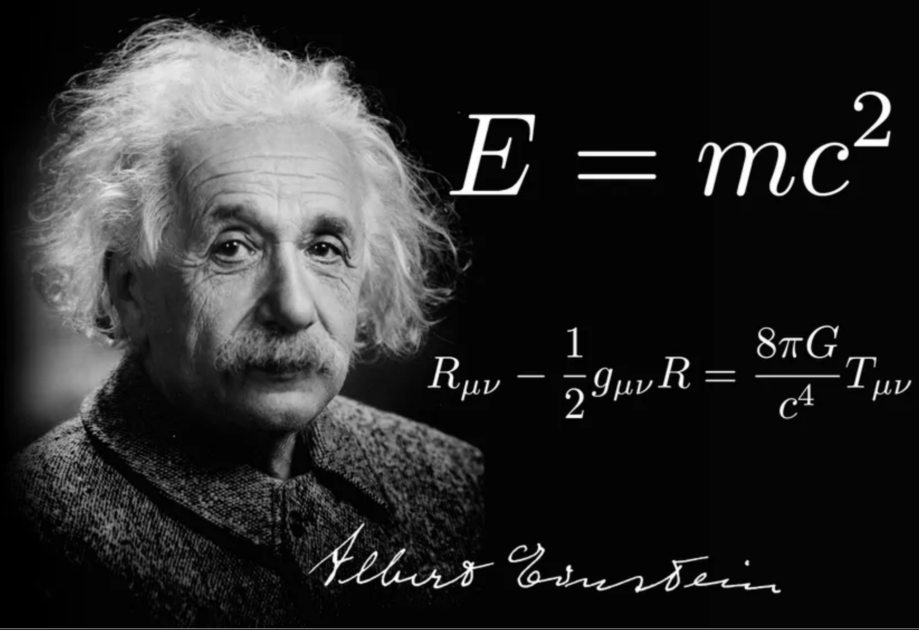
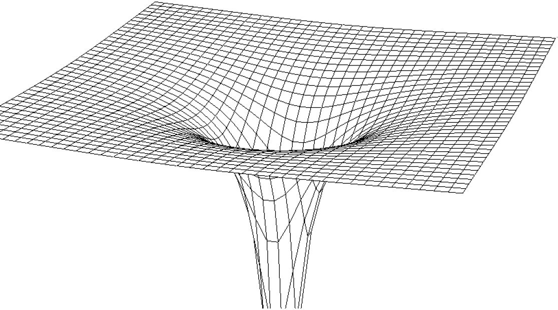
General Relativity



Schwarzschild Black Holes

Schwarzschild 1916

- solution of the Einstein equations in vacuum



Karl Schwarzschild 1873 — 1916

- Schwarzschild radius

$$r_{\mathrm{H}} = \frac{2GM}{c^2}$$

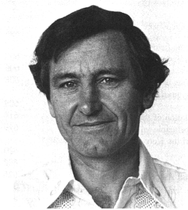
- event horizon
- singularity $r = 0$

- black hole with mass M

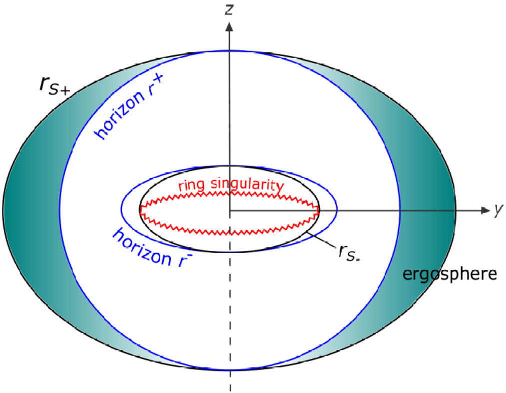
Kerr Black Holes

rotating generalization of the Schwarzschild black holes:
Kerr (1963)

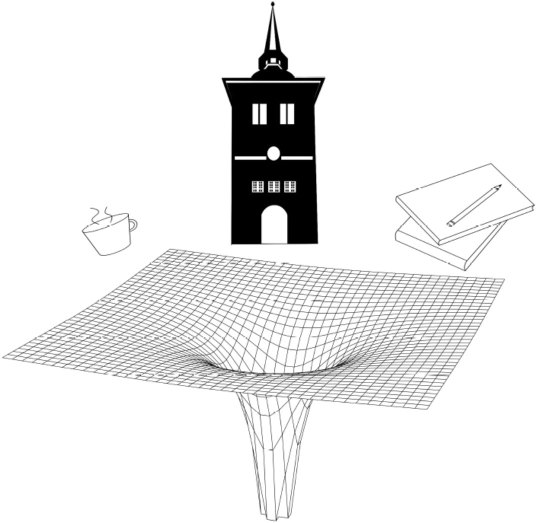
astrophysical black holes
are described by the Kerr solution!?



Roy Kerr *1934

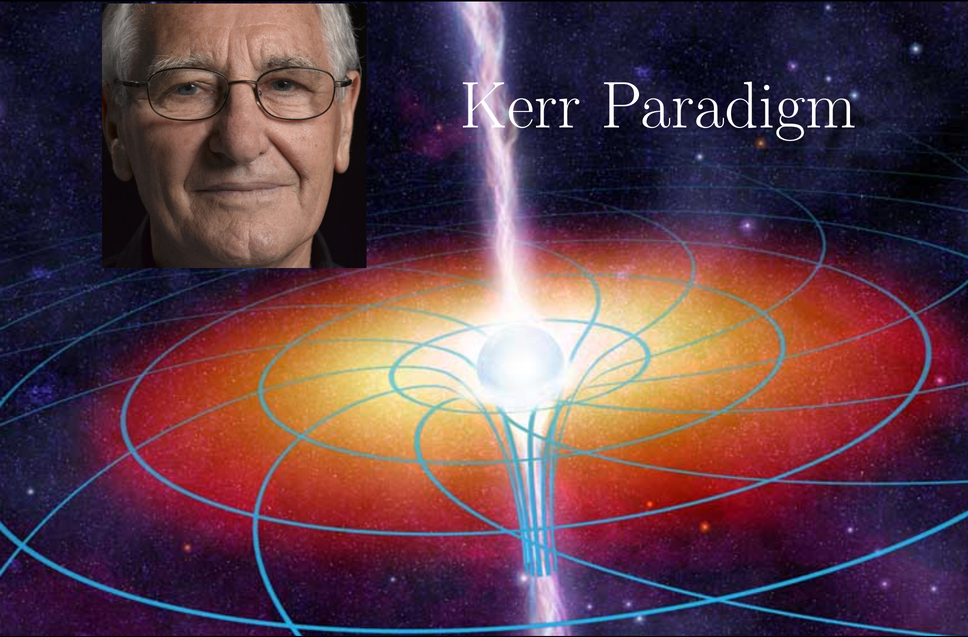


Kerr Black Holes

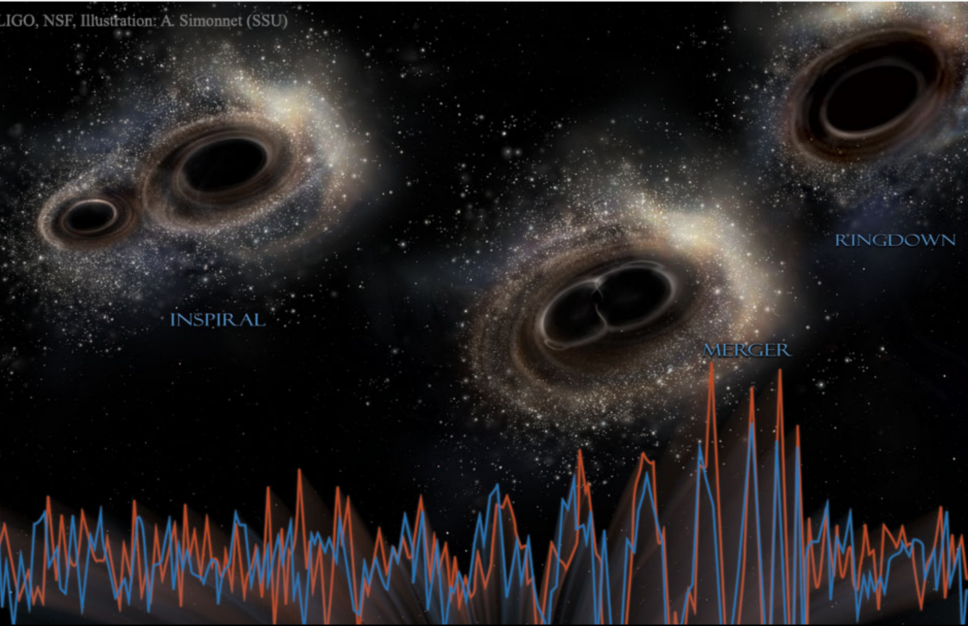


Black holes have no hair

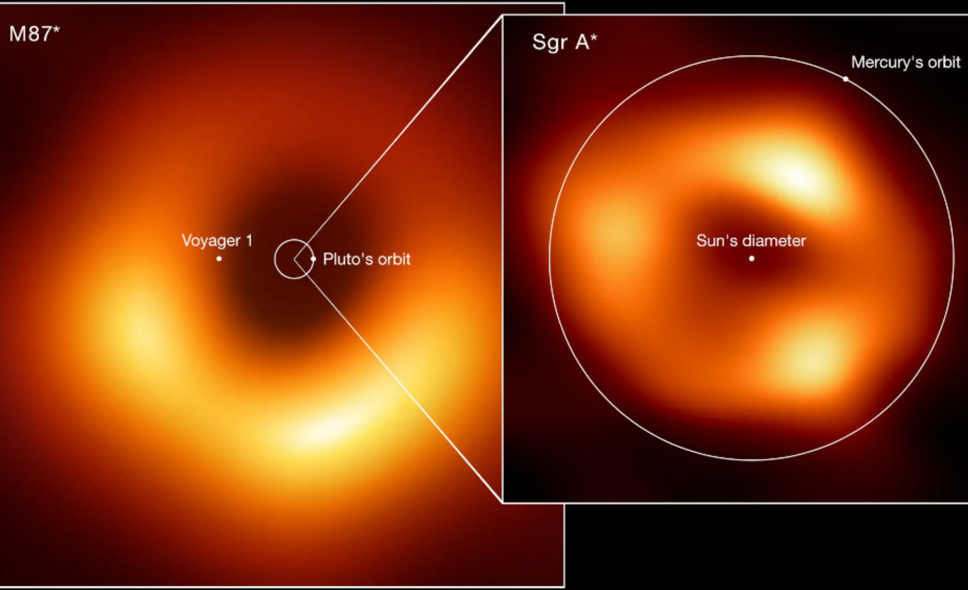
Testing GR: Kerr black holes



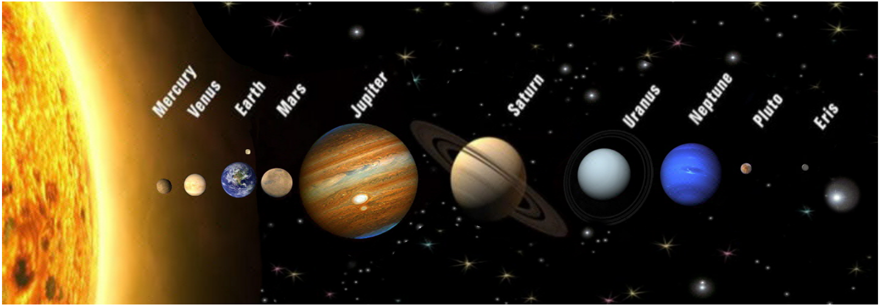
Testing GR: Kerr black holes



Testing GR: Kerr black holes



Alternative Theories of Gravity



- Compatible with all solar system tests!
- Strong gravity?
  - Black holes
  - Neutron stars
  - Exotic compact objects
- Cosmology?

Outline

Einstein-scalar-Gauss-Bonnet Theories

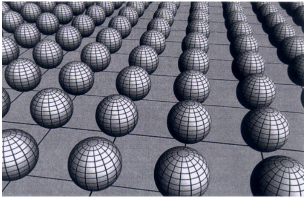
EsGB action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} (\partial_\mu \varphi)^2 + f(\varphi) R^2_{\rm GB} \right]$$

Gauss-Bonnet term: quadratic in the curvature

$$R^2_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

coupling function $f(\varphi)$



The resulting set of equations of motion are of second order (Horndeski).

EdGB black holes: properties

Kanti et al. hep-th/9511071, Torii et al. gr-qc/9606034

dilatonic coupling function

$$f(\varphi) = \frac{\alpha}{4} e^{-\gamma\varphi}$$

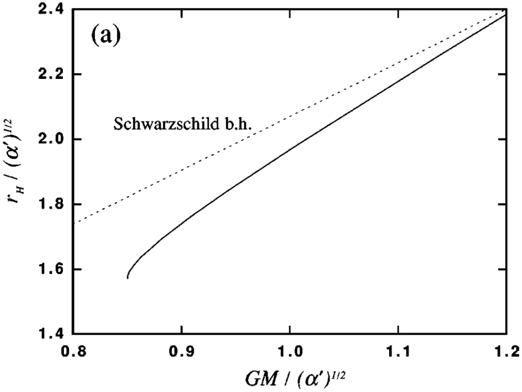
static black holes

critical black holes:

horizon expansion

$$\sqrt{1 - 6\frac{\alpha'^2}{r_h^4} e^{2\varphi_h}}$$

lower bound
on the horizon size
for fixed α'

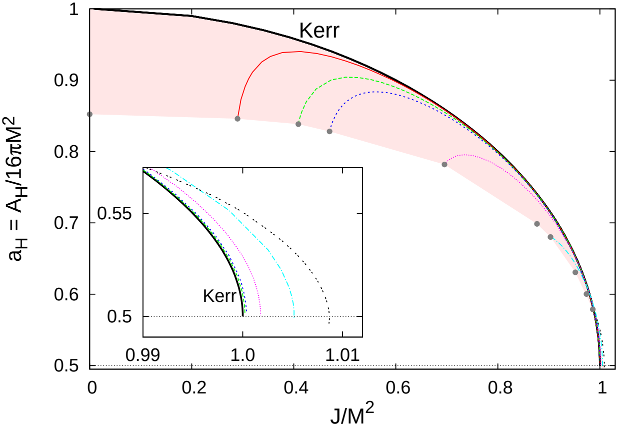


lower bound on the mass

EdGB black holes: properties

Kleihaus et al. 1101.2868

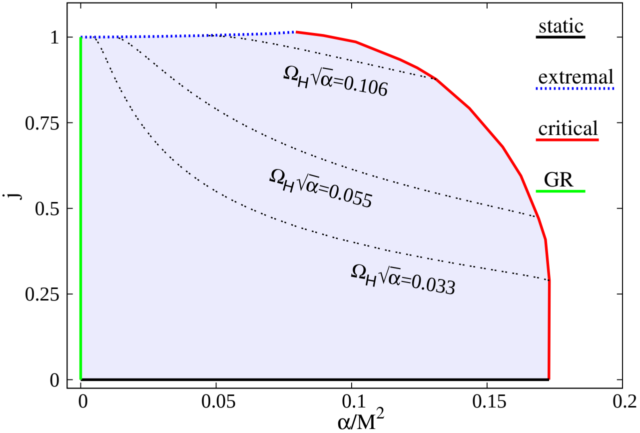
horizon area versus angular momentum



EdGB black holes: properties

Blazquez-Salcedo et al. arXiv: 2407.20760

angular momentum versus coupling constant

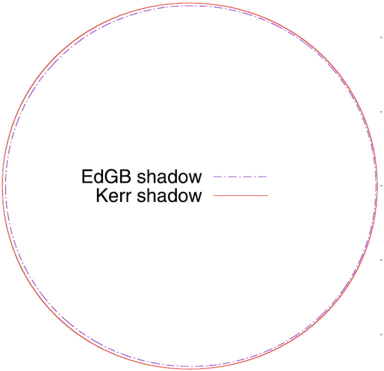


limited domain of existence of background black hole solutions

EdGB black holes: properties

Cunha et al. arXiv:1701.00079

shadow



$$\alpha/M^2 = 0.172, \;\; J/M^2 = 0.41$$

Outline

EdGB black holes: QNMs of static BHs

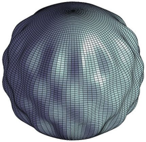
perturbation theory: damped oscillations

metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(r) + \epsilon h_{\mu\nu}(t, r, \theta, \phi)$$

scalar

$$\phi = \varphi_0(r) + \epsilon \delta\varphi(t, r, \theta, \phi)$$



polar modes: even-parity perturbations

axial modes: odd-parity perturbations (pure space-time modes)

master equation: Schrödinger-like equation

eigenvalue ω

$$\omega = \omega_R + i\omega_I$$

frequency: ω_R

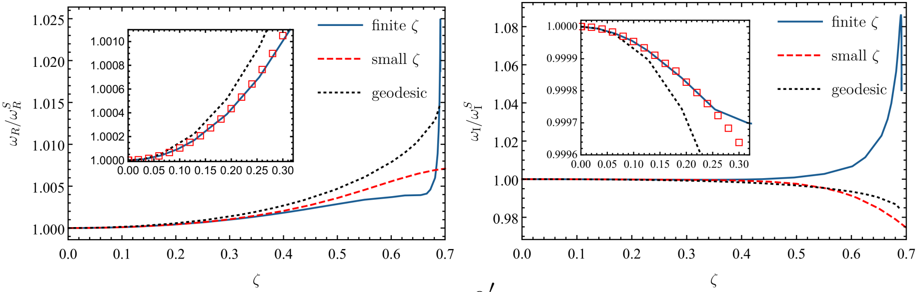
decay time: $\tau = 1/\omega_I$

EdGB black holes: QNMs of static BHs

Blazquez-Salcedo et al. arXiv:1609.01286

quasi-normal mode (axial $l = 2$) versus coupling constant

normalized to the Schwarzschild values



real part $\qquad \zeta = \dfrac{\alpha'}{M^2} \qquad$ imaginary part

EdGB black holes: QNMs of static BHs

Blazquez-Salcedo et al. 1609.01286

quasi-normal mode (polar $l = 2$) versus coupling constant

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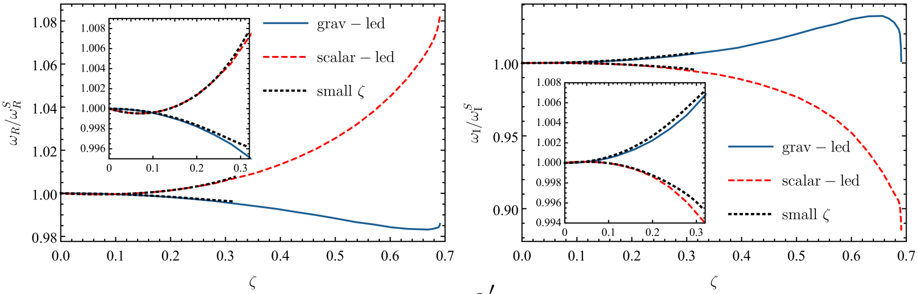

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EdGB black holes: QNMs of rotating BHs

axial-led (A) perturbations

$$\delta h_{\mu\nu}^{(A)} = e^{i(M_z\phi-\omega t)} \begin{pmatrix} 0 & 0 & a_1(r,\theta) & a_2(r,\theta) \\ 0 & 0 & a_3(r,\theta) & a_4(r,\theta) \\ a_1(r,\theta) & a_3(r,\theta) & 0 & 0 \\ a_2(r,\theta) & a_4(r,\theta) & 0 & 0 \end{pmatrix}$$

polar-led metric (P) perturbations

$$\delta h_{\mu\nu}^{(P)} = e^{i(M_z\phi-\omega t)} \begin{pmatrix} N_0(r,\theta) & H_1(r,\theta) & 0 & 0 \\ H_1(r,\theta) & L_0(r,\theta) & 0 & 0 \\ 0 & 0 & T_0(r,\theta) & 0 \\ 0 & 0 & 0 & S_0(r,\theta) \end{pmatrix}$$

scalar-led perturbations

$$\varphi \;\; = \;\; \varphi^{(bg)} + \epsilon\delta\varphi(t,r,\theta,\phi) = \varphi^{(bg)} + \epsilon e^{i(M_z\phi-\omega t)}\Phi(r,\theta)$$

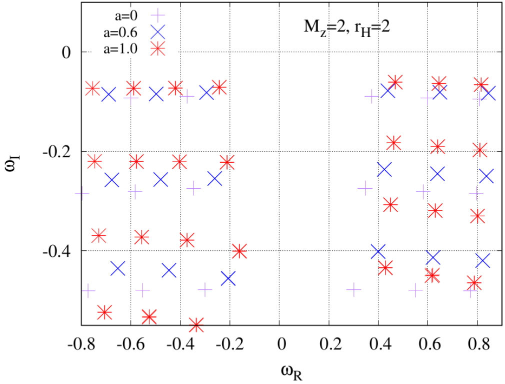
time dependence $e^{-i\omega t}$, ϕ-dependence $e^{iM_z\phi}$, no r - θ decoupling

EdGB black holes: QNMs of rotating BHs

approaches:

- perturbation theory for slow rotation
- numerical methods for rapid rotation
  - spectral decomposition
    r in Chebyshev polynomials, θ in Legendre functions

Kerr QNMs



EdGB black holes: QNMs of rotating BHs

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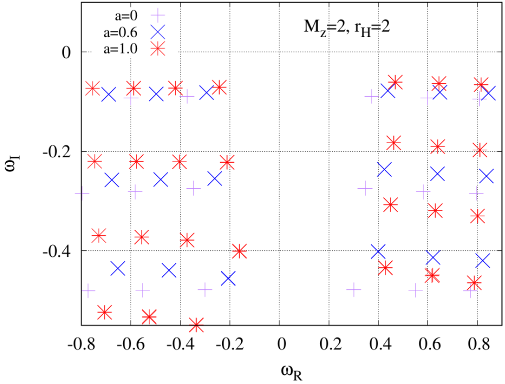
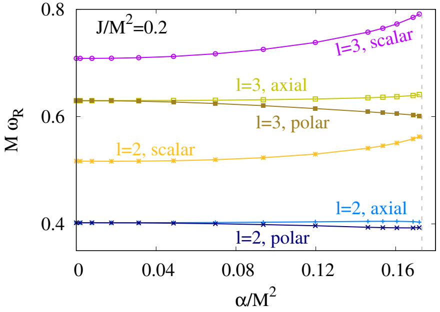


EdGB black holes: QNMs of rotating BHs

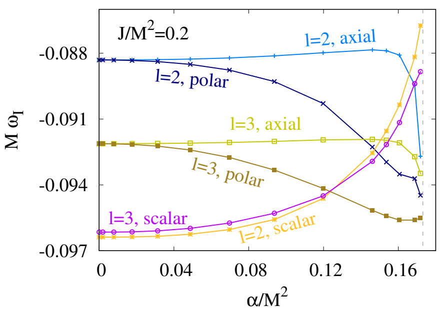
Blazquez-Salcedo et al. arXiv: 2407.20760, 2412.17073

polar-led, axial-led, and scalar-led QNMs versus coupling constant α/M^2

$$J/M^2 = 0.2$$



real part



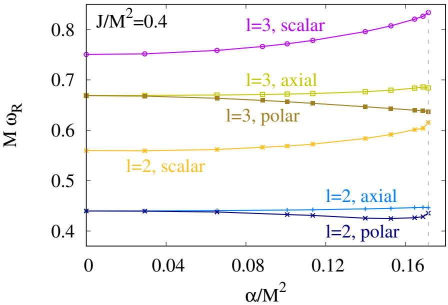
imaginary part

EdGB black holes: QNMs of rotating BHs

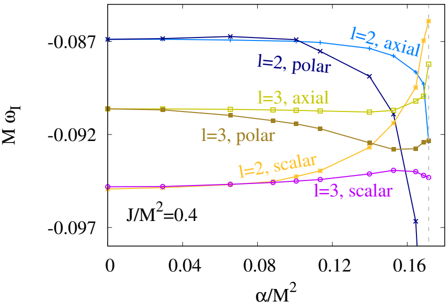
Blazquez-Salcedo et al. arXiv: 2407.20760, 2412.17073

polar-led, axial-led, and scalar-led QNMs versus coupling constant α/M^2

$$J/M^2 = 0.4$$



real part



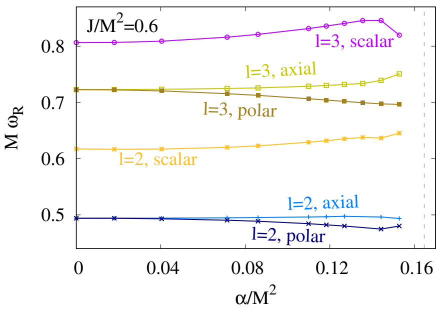
imaginary part

EdGB black holes: QNMs of rotating BHs

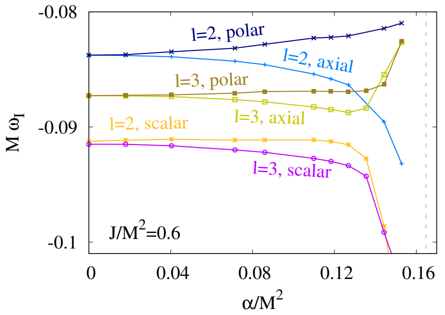
Blazquez-Salcedo et al. arXiv: 2407.20760, 2412.17073

polar-led, axial-led, and scalar-led QNMs versus coupling constant α/M^2

$$J/M^2 = 0.6$$



real part



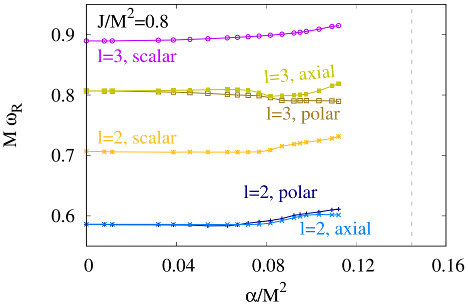
imaginary part

EdGB black holes: QNMs of rotating BHs

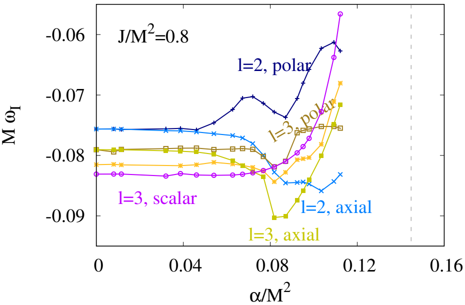
Blazquez-Salcedo et al. arXiv: 2407.20760, 2412.17073

polar-led, axial-led, and scalar-led QNMs versus coupling constant α/M^2

$$J/M^2 = 0.8$$

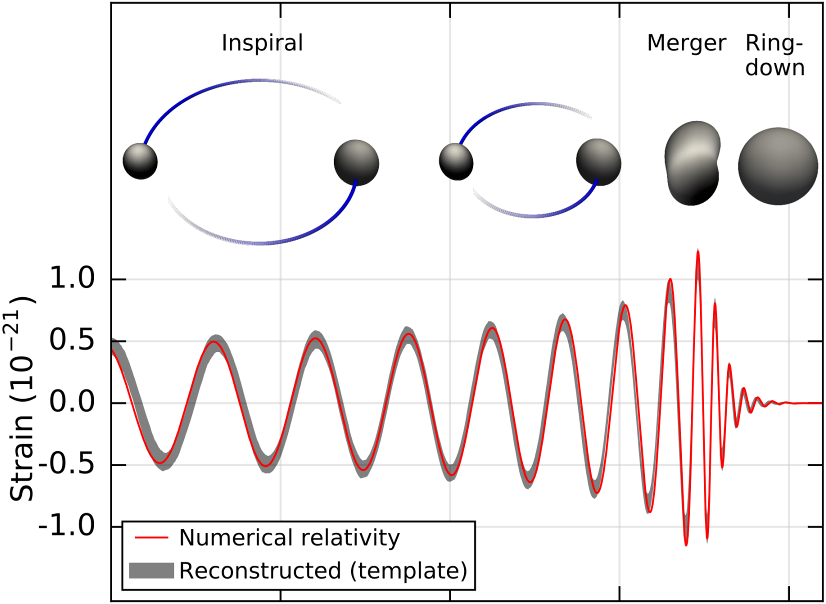


real part



imaginary part

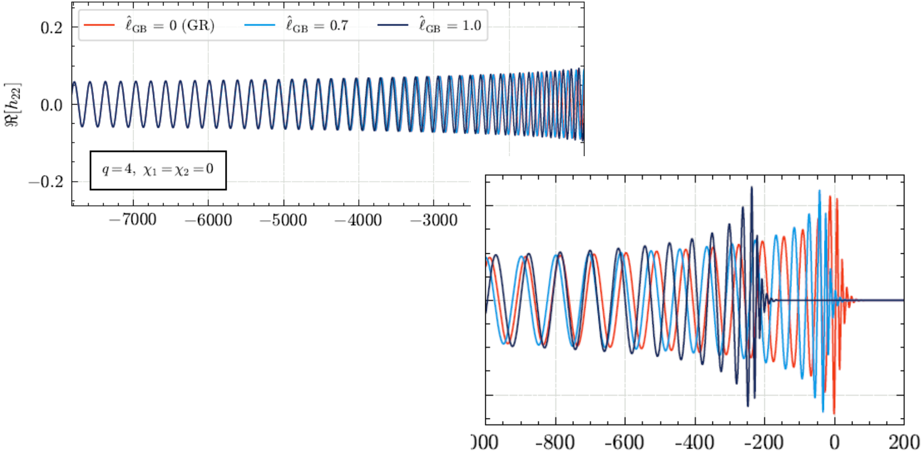
Inspiral, merger, ringdown



EdGB black holes: effective-one-body formalism

Julié et al. 2406.13654

Inspiral-merger-ringdown waveforms in EdGB



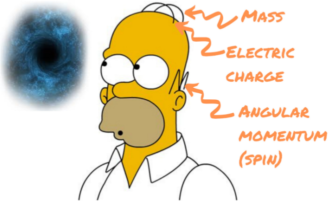
Outline

Conclusions

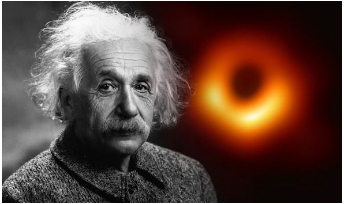
GR versus generalized gravity theories

GR black holes: no hair

- Kerr
  - GW: QNMs, Numerical Rel.
  - shadow
  - ...



black holes beyond GR: hair



- EdGB
  - GW: QNMs, EOB approach
  - perturbatively stable
  - shadow
- ...

THANKS