QCD strings and effective string theories.

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Figure: Prof A.N. Mitra – he opened our eyes to the magical world of elementary particles.

Figure: The unforgettable stage..

Quarks,Mitra, and Me

- The quark model was proposed independently by Gell-Mann and Zweig in 1964.
- For my MSc Thesis in 1966, Mitra suggested working on The Meson and Baryon decays in the quark model.
- He was clearly among the few worldwide that took the quark model seriously right from its inception.
- The year after, in 1967, I went to UCSB, barely 100 Kms from CALTECH where the quark model was born.
- While writing my recent book Strings to Strings I discovered a Mitra-UCSB connection: the paper by Mitra, Gillespie, Sugar and Nargis in 1965 on four-particle scattering.
- No one took the quark model seriously there.
- Feynman even made fun of them calling it the Quack Model!

Quarks....

- The quark model was an Outgrowth from Sakata Model, Eightfoldway, SU(3)..
- Incidentally, the spectacular prediction of Ω^- owes it to the Eightfoldway and not to the quark model as is sometimes claimed erroneously!
- To avoid anti-baryons in building baryons as in the Sakata Model, both Gell-Mann and Zweig introduced fractional baryon number and electric charge for the quarks.
- Almost immediately, the quark model was seen to suffer from the so-called statistics problem
- In 1965, Han and Nambu came up with an alternate quark model which not only proposed ways of solving the statistics problem (they did not call it color), but also paved the way for Quark Dynamics through gauging.

Han-Nambu Quarks

- In hindsight, it was a monumental opportunity missed that we did not study the Han-Nambu paper more closely!
- No less than Gell-Mann himself had this to say about that work:
  If I had read Nambu's paper it would have set me ahead by several years and Nambu then invented what amounted to QCD, or, the beginning of QCD.

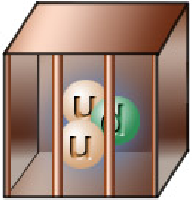
The birth of QCD..

- The attractive feature of Han-Nambu quarks was their integral electric charges unlike the fractional charges of the Gell-Mann-zweig quarks.
- But the heavy price to pay for this was an unavoidable mixing of flavour with colour.
- It took almost a decade for the correct identification of color in the Gell-Mann-Zweig quark model.
- It was supremely elegant and completely avoided any color-flavour mixing!
- It was identified as the fundamental representation of a New SU(3), the color group.
- Quark Dynamics was realized through the Yang-Mills theory of the color group.
- Rather ironically, even to this day there are no acid tests to distinguish the Han-Nambu and Gell-Mann-Zweig theories!
- Extensive works by G.Rajasekaran and collaborators..

The other track..

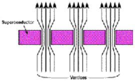
- Not everyone subscribed to the quark model for a variety of reasons.
- Even QFT itself had its critics who championed the S-matrix approach pioneered by Heisenberg.
- After a decade long, rather impressive line of developments like Dispersion Theory, Dual Resonance Models etc., Relativistic Strings were proposed in 1970 by Nielsen, Nambu and Susskind to explain many features of strongly interacting particles called hadrons that could not, at that point, be explained by any Relativistic Quantum Field Theory.
- This idea also ran into several difficulties like requiring space-time dimensions to be 26 etc., and was abandoned by many in favor of QCD.
- The punch-line of our work is the emergence of strings within QCD itself!

Confinement problem!



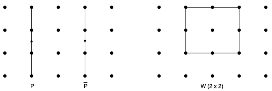
- Major unsolved problem for QCD: Quark Confinement.
- Simply stated, no free quarks had been observed even during very energetic collisions between hadrons.
- That would not be so if hadrons were bound states of a QFT.
- What should have been a theoretical embarassment was turned into a virtue with the idea of Permanent Quark Confinement.
- But it was far from clear if any QFT, leave alone QCD, could ever possess that property!

QCD Vacuum a (Dual)Superconductor?



- Imagine a monopole-antimonopole pair immersed in a superconductor.
- The superconductor would tend to expel magnetic flux everywhere! But flux conservation will not allow flux to be expelled everywhere!–Ajit's liquid oxygen!
- A superconducting ground state would squeeze magnetic flux into a very thin Flux Tube with constant energy per unit length.
- The thin flux tube would confine forever a magnetic monopole-antimonopole pair!
- For color electric charges to be confined one needs a dual superconductor.
- An added bonus: At some sufficiently high temperature, supercoonductivity disappears leading to deconfinement!

Numerical Studies of QCD Flux Tubes.



- Flux tubes can be investigated in Lattice Gauge Theory by studying either very large Wilson Loops or Polyakov Loop Correlators.
- The central object of interest is the static quark-antiquark potential

$$\langle\, W_{R\times T}\rangle \simeq\ e^{-T\,V(R)} \qquad \langle\, P(0)P^{*}(R)\rangle \simeq\ e^{-T\,V(R)}$$

- For large separations

$$V(r) = \sigma\, r + \frac{A}{r} + \frac{B}{r^2} + \frac{C}{r^3} + \ldots$$

- $\sigma \neq 0$ signals linear confinement.
- On rather general grounds, $B = 0$
- $A = \frac{D-2}{24\,\pi}$. This is the well-known Lüscher term, not to be confused with Coulomb Potential.
- A convenient dimensionless quantity is $c(r) = \frac{r^3}{2} \cdot V''(r)$.

$$c(r) = A + \frac{6C}{r^2} + \ldots$$

Numerical Studies of QCD Flux Tubes.

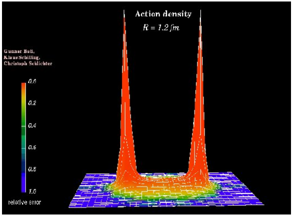
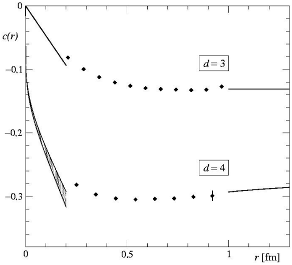


Figure: Flux-tube profile by Gunnar Bali et al..

- There is overwhelming lattice evidence for linear confinement which manifests as Area Law for Wilson Loops.
- Even the profile of flux-tubes has been accurately studied on lattice.

Numerical Studies: Lüscher-Weisz.

- A breakthrough in these studies came from an amazing algorithm by Lüscher and Weisz called multilevel
- With this, the A term, also called the Lüscher term could be estimated to about 15% accuracy.
- LW stopped short of exploring the most critical region. As a result, nothing definitive could be said about the string-like behaviour of flux tubes.



KABRU

- In 2004 as part of the tenth plan, as well as the objectives of an alliance of Indian Lattice Theorists(LGTgatabandhan) I built a teraflop Linux cluster KABRU). It was on the Top 500 Supercomputers of The World twice in a row.
- I was looking for a worthy problem to put KABRU to work.

Numerical Studies: Works with Pushan Majumdar.

- Pushan Majumdar(who passed away at a very young age in 2020) by that time had become an expert on Multilevel and had applied it to a number of interesting and fundamental problems.
- We decided to do a very thorough study of flux tubes in d=3,4 with SU(2) and SU(3).
- Utilising all the design features of KABRU fully, we could handle much bigger lattices.
- Most importantly we could achieve significantly higher accuracies and probe much larger $Q-\bar{Q}$ separations.

Numerical Studies: Works with Pushan.

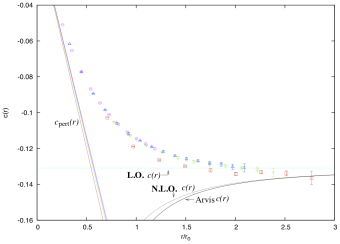


Figure: d=3 SU(2)

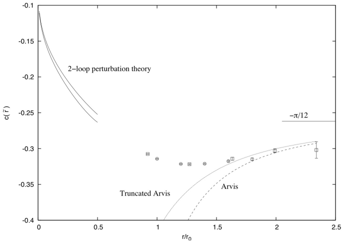


Figure: d=4 SU(3)

- Continuum limit reached for d=3. Beyond 1 loop, infrared divergent.
- Our results can be used for accurate determination of IR counterterms
- In both d=3,4 clear convergence to string domain.
- Smooth interpolation from Asymptotic Freedom to Confinement

Flux Tubes as Fundamental strings?

- The exact potential(ground state energy) for Bosonic Strings fixed at the end-points is the Arvis Potential:

$$V_{arvis} = \sqrt{\sigma^2 r^2 - \frac{D-2)\pi\sigma}{12}}$$

For large r

$$V_{arvis} \to \sigma r - \frac{(D-2)\pi}{24}\frac{1}{r} + \frac{(D-2)^2\pi^2}{8.144.\sigma}\frac{1}{r^3} + \ldots$$

Our measured values approached this quite well already around $r = 1$ fm.

- These studies indicated that to a high degree flux tubes actually behaved like fundamental strings!
- But bosonic strings are consistent only in 26 dimensions but QCD is a consistent theory in $D = 3, 4$ dimensions.
- A numerological accident? Extremely unlikely!!

Polchinski-Strominger Idea

- Motivated by Polyakov subcritical string theories Polchinski-Strominger proposed:
- Modify the action for string theory from

$$S_0 = \frac{1}{4\pi a^2} \int d\tau^+ d\tau^- \quad \partial_+ X \cdot \partial_- X$$

- to

$$S = S_0 + \frac{\beta}{4\pi} \int d\tau^+ d\tau^- \frac{\partial_+ L \cdot \partial_- L}{L^2}$$

where

$$L = \partial_+ X \cdot \partial_- X$$

Consequences

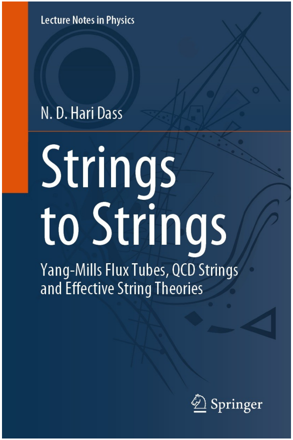
- Such String theories become consistent in all dimensions if $\beta = \frac{D-26}{12}$.
- The action is sick for many configurations but remains sensible for small fluctuations around a straight(confining) string configuration.
- Can be used as an effective theory. It is a higher time derivative theory.
- The actions are invariant under the local transformations

$$\delta_{\pm} X^{\mu} = -\epsilon^{\pm}(\tau^{\pm})\, \partial_{\pm} X^{\mu}$$

Various Results

- With Peter Matlock I analytically proved in 2006 that to order R^{-3} the spectrum of these theories is the same as that of Bosonic String theory(Arvis).
- With Peter Matlock I developed a Covariant Calculus with which Effective String Theories can be constructed to arbitrary order.
- With Peter and my ex-student Yashas Bharadwaj(CHEP) we proved a number of results which are beyond the scope of this talk.

An 'Effective' Bibliography



Not a hint to buy my book!: Effective String Theories of Yang-Mills Flux Tubes. arXiv:2312.10629[hep-th]